# DIFFRACTION RADIATION WHICH ACCOMPANIES THE MOTION OF CHARGED PARTICLES NEAR AN OPEN RESONATOR

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Assuming that the current is given, we consider the radiation that occurs during the motion of charged particles near an open hollow ideally conducting cylinder. It is shown that the particle causes an outburst of radiation with a narrow effective frequency spectrum. We study the directionality diagram (radiation pattern) and the total energy of the resonant radiation.

The necessity of optimization of output parameters of the generators of diffraction radiation continues to stimulate the study of diffraction lattices of various forms [1]. For example, Veliev et al. [2] studied the radiation of particles which pass above a lattice of resonant elements (open hollow metal cylinders) and it was established that the radiation of charged particles above this lattice is more efficient than, e.g., above a comb or an echelette grating.

Since the properties of individual elements form the basis for the properties of the lattice, the properties of individual elements should be studied in more detail. In addition, a hollow cylindrical resonator with a longitudinal slot is in itself very close to the model of a real resonator of uhf devices, e.g., the resonator element of the magnetron.

## 1. FORMULATION OF THE PROBLEM AND CONSTRUCTION OF THE FORMAL SOLUTION

Assuming that the current is given, we consider the diffraction radiation which accompanies the motion of a planar infinite electron current with the charge density

$$\rho = \rho_0 \delta (y - p) \exp [i (kx/\beta - \omega t)] \tag{1}$$

near a hollow circular cylinder with a longitudinal slot (Fig. 1). Here $\omega$ and $\rho_0$ are the frequency and amplitude of the current modulation; $k = \omega/c$, $\beta = v/c$, relative velocity of motion of the current; $p = a + h$; and h, impact parameter. The cylinder is assumed infinitely thin and ideally conducting.

We note that the problem of radiation of the current (1) can be viewed as a problem for individual spectral components of the radiation of an unmodulated beam, or a problem for a single particle, or more exactly, for its two-dimensional analogue, i.e., a charged wire [3].

It is known that the proper electromagnetic field of the current (1) has a form of a slow inhomogeneous plane surface wave [1] whose only nonzero component of the magnetic field is equal to

$$H_z^0 = 2\pi\rho_0\beta \exp(-q \mid y - p \mid + ikx/\beta) \mid y - p \mid /(y - p), \tag{2}$$

where $q = k\gamma\beta^{-1}$, $\gamma = (1 - \beta^2)^{1/2}$, and the factor $\exp(-i\omega t)$ has been omitted. In a cylindrical coordinate system, the field (2) can be written in the following way:

$$H_z^0 = 2\pi\rho_0\beta e^{-qp} \sum_{n=-\infty}^{\infty} I_n(kr)[i(1-\gamma)/\beta]^n e^{in\varphi}. \tag{3}$$

In the fixed-current approximation, the study of the diffraction radiation reduces to the determination of the electromagnetic field $H_z^S$ which occurs as a result of the scattering of (2) from the open cylinder. The function $H_z^S(r, \varphi)$ satisfies the known conditions (see, e.g., [2]), and can be written in the following form:

$$H_z^s = -2\pi\rho_0\beta e^{-qp} \sum_{n=-\infty}^{\infty} \mu_n \left\{ \begin{matrix} I_n'(ka) H_n^{(1)}(kr) \\ H_n^{(1)'}(ka) I_n(kr) \end{matrix} \right\} e^{in\varphi} \quad \begin{matrix} (r \geqslant a) \\ (r \leqslant a) \end{matrix}, \tag{4}$$

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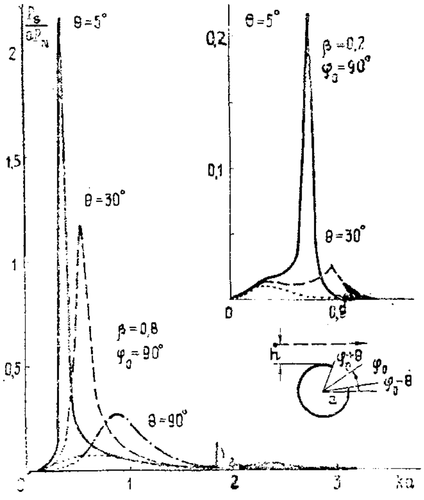

Fig. 1

where the quantities $\mu_n$ are the Fourier components of the surface current density induced on the cylinder which satisfy a system of equations of the first kind in terms of paired series (with a trigonometric kernel) of the same type as in the problem of diffraction of a plane wave (see, e.g., [4]). The allowed solutions of these equations belong to the class $\tilde{l}^2$, where $\sum_n |\mu_n|^2 |n| < \infty$, which follows from the condition that the energy at the sharp edges of the open cylinder is finite. The regularization of the paired equations can be achieved by inverting the statistical part of the appropriate operator by the method of the conjugation problem [5], and leads to a system of algebraic equations of the second kind:

$$\mu_m + \sum_{n=-\infty}^{\infty} A_{mn}\mu_n = B_m, \quad m = 0, \pm 1, \ldots, \tag{5}$$

where

$$A_{mn} = \Delta_n T_{mn}, \quad \Delta_n = |n| + i\pi (ka)^2 I_n'(ka) H_n^{(1)\prime}(ka),$$

$$T_{mn} = (-1)^{m+n} e^{i(m-n)\varphi_0} \begin{cases} V_{n-1}^{-1}(-\cos\theta), & m = 0 \\ V_{m-1}^{n-1}(-\cos\theta)/m, & m \neq 0 \end{cases},$$

$$B_m = i\pi (ka)^2 D e^{-qa} \sum_{n=-\infty}^{\infty} I_n'(ka)\,[i(1-\gamma)/\beta]^n e^{in\varphi_0},$$

$$D = -2\pi\rho_0\beta e^{-qh},$$

and the quantities $V_{m-1}^{n-1}$ are defined in [5].

We note that a formal solution analogous to (5) was first given in [6]. The solution is rigorous in the sense that Eqs. (5) can be solved with any given accuracy.

## 2. RADIATION OF PARTICLES MOVING NEAR A CIRCULAR CYLINDER

System (5) allows a transition to the limiting case $\theta = 0$ which corresponds to the case of radiation by a particle moving near a closed circular ideally conducting cylinder. In this case, Eq. (5) degenerates into a set of equalities for the quantities $\mu_n$, and the result can be written down explicitly. For example, the current induced on the cylinder is equal to

$$j_0(\varphi) = cDe^{-qa}(i2\pi ka)^{-1} \sum_{n=-\infty}^{\infty} [i(1-\gamma)/\beta]^n e^{in\varphi} [H_n^{(1)\prime}(ka)]^{-1}. \tag{6}$$

For long wavelengths ($ka < \beta$) the current density can be represented in the form of the asymptotic series

$$j_0(\varphi) = cDe^{-qa}(4\pi)^{-1}[1 + 2ka\beta^{-1}(\gamma \sin\varphi + i\cos\varphi)]\,[1 + O(k^2a^2\beta^{-2})]. \tag{7}$$

It is not difficult to use (7) to verify that the function $|j_0(\varphi)|$ reaches a minimum at the point $\varphi = -90°$ and a maximum at the point $\varphi = 90°$. The extremum values are proportional to $1 \mp 2ka\gamma\beta^{-1}$, so that in this approximation, the

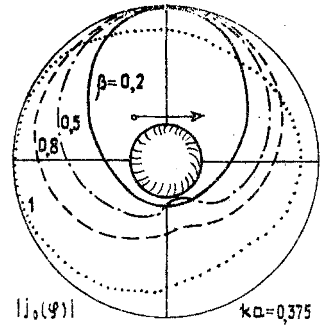


Fig. 2. Current distribution at the surface of the circular cylinder.

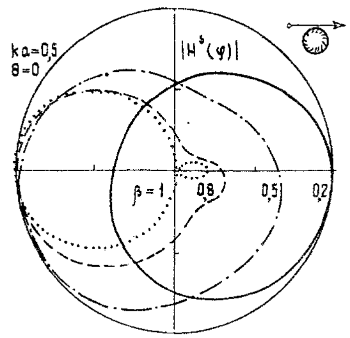


Fig. 3

minimum can be equal to zero. In other words, in the scattering of the surface wave (2) from a closed circular cylinder, the points $\varphi = \pm 90°$ play the role of the "light" and "dark" poles, by analogy with the points $\varphi = 0$, $180°$ in the diffraction of plane homogeneous wave. This result is confirmed by the numerical summation of series (6) using a computer whose results are shown in Fig. 2.

The directionality diagrams (radiation patterns) of the diffraction radiation for a closed cylinder (Fig. 3) display a weak directionality. The total power of the radiation is obtained by evaluating the series

$$P_s^0 = cD^2 (2\pi k)^{-1} e^{-2qa} \sum_{n=-\infty}^{\infty} |\mu_n I_n'(ka)|^2 \tag{8}$$

and is in the long-wavelength case proportional to $\omega^3$:

$$P_s^0 = (1/8)\, c\pi^3 \rho_0^2 (2 + \beta^2)\, k^3 a^4 \exp(-2qp)\, [1 + O(k^2 a^2 \beta^{-2})]. \tag{9}$$

The frequency dependence of $P_s^0$ is shown in Fig. 1 by the dashed line.

## 3. UNIFORM ASYMPTOTIC BEHAVIOR AT LONG WAVELENGTHS. RESONANT DIFFRACTION RADIATION

We now return to the case of radiation of a particle moving near an open resonator with a coupling slot. It can be shown that an estimate of the norm of the matrix element (5) gives the following inequality which is valid in the long-wavelength region:

$$g = \max_{m \neq 0} \left[\sum_n |A_{mn}| / (1 - A_{mm})\right] < (ka)^2 C(\varphi_0, \theta).$$

For sufficiently small $ka$, system (5) can therefore be solved by iteration. The solution of (5) can be represented in the form of an asymptotic series which is uniform with respect to the parameters $\theta$ and $\varphi_0$. In the zero approximation, with accuracy up to term $[1 + O(k^2 a^2 \beta^{-2})]$,

$$\mu_{\pm 1} = -D(ka)^2 \cos^2\frac{\theta}{2} \left\{\mu_0 (1 \pm \gamma)\beta^{-1} \exp(\pm i\varphi_0) + \frac{\pi}{2}\left[1 + \sin^2\frac{\theta}{2} \exp(\mp 2i\varphi_0)(1 \pm \gamma)^2 \beta^{-2}\right]\right\},$$

$$\mu_0 = Di\pi (ka)^2 \{4(ka)^2 - 4(k_0 a)^2 + (ka)^4 [i\pi - 2(k_0 a)^2 \xi(\theta)]\}^{-1} \times$$

$$\times \left\{ka\left[2 - (k_0 a)^2 \left(1 - 4\sin^2\frac{\theta}{2} + 3\sin^4\frac{\theta}{2}\right)\beta^{-2}[(1 + \gamma^2)\cos 2\varphi_0 - \right.\right. \tag{10}$$

$$\left.\left. - 2i\gamma \sin 2\varphi_0] + 4i(k_0 a)^2 \beta^{-1} \cos^2\frac{\theta}{2}(\cos\varphi_0 - i\gamma \sin\varphi_0)\right]\right\},$$

where $(k_0 a)^2 = \left(-2\ln\sin\frac{\theta}{2}\right)^{-1}$, $\xi(\theta) = \cos^4\frac{\theta}{2} + \frac{1}{4}\sum_{n=1}^{\infty}\frac{1}{n(n+1)}[P_n(\cos\theta) + P_{n-1}(\cos\theta)]^2$, and $P_n$ is the Legendre polynomial.

Expressions (10) make it possible to obtain the following formula for the total energy of the diffraction radiation ($\mu_0 = \mu_0' + i\mu_0''$):

$$P_s = \frac{c}{8\pi} D^2 e^{-2qa} ka^2 \left\{ |\mu_0|^2 \left[ 1 + 2(ka)^2 \beta^{-2} \cos^4 \frac{\theta}{2} (2-\beta^2) + \right.\right.$$

$$+ \frac{\pi}{2} (ka)^4 \cos^4 \frac{\theta}{2} \left] \left[ 1 + \sin^4 \frac{\theta}{2} \beta^{-4} (8 - 8\beta^2 + \beta^4) + 2 \sin^2 \frac{\theta}{2} \beta^{-2} \times \right.\right.$$

$$\times (2-\beta^2) \cos 2\varphi_0 \right] + 2\pi (ka)^2 \beta^{-1} \cos^4 \frac{\theta}{2} \mu_0' \cos\varphi_0 \left[ 1 + \right. \tag{11}$$

$$\left. + \sin^2 \frac{\theta}{2} \beta^{-2} (4 - 3\beta^2) \right] + 2\pi (ka)^2 \beta^{-1} (1-\beta^2)^{1/2} \cos^4 \frac{\theta}{2} \; \mu_0'' \sin\varphi_0 \left[ 1 - \sin^2 \frac{\theta}{2} \beta^{-2} (4-\beta^2) \right] \right\}.$$

For $\theta \to 0$, formula (11) reduces to the analogous expression (9) for the circular cylinder. If $\theta \to 180°$, Eq. (11) corresponds to the radiation of a particle moving above a planar strip of width $2a(\pi - \theta)$:

$$P_s = (1/4) c\pi D^2 e^{-2qa} k^3 \left( a \cos \frac{\theta}{2} \right)^4 \beta^{-4} [\cos^2 \varphi_0 + (1-\beta^2)(1 + \sin^2\varphi_0 - \beta^2 \sin^2 \varphi_0)]. \tag{12}$$

Formula (11) encompasses the region of Rayleigh scattering when $k \ll k_0 < \beta$ and $P_s = O(k^3 a^4)$, as well as the resonant scattering at a frequency equal to $k_0$ for small $\theta$. It is known that the latter case corresponds to the excitation of a quasiproper slot oscillation which can be conditionally denoted by $H_{00}$ [4].

In resonant conditions, the efficiency of radiation of charged particles sharply increases:

$$P_s^{res} = \frac{2\pi c}{k_0} \rho_0^2 \beta^2 e^{-2q\rho} \left( 1 + 2 k_0 a \frac{1}{\beta} \cos^2 \frac{\theta}{2} \sin\varphi_0 \right) [1 + O(k^2 a^2 \beta^{-2})]. \tag{13}$$

This is considerably larger (by four orders of magnitude in frequency) than the energy (9) of radiation which accompanies the motion of a charged wire near a planar ideally conducting cylinder: $P_s^{res}/P_s^0 = O(k^{-4} a^{-4})$.

The resonant increase of the radiation energy can be expressed in terms of the increase of the effective dimensions of the scattering object. By equating (9) and (13) for fixed values of parameters $\beta$, h, k, $\rho_0$, and $\varphi_0 = 90°$ we arrive at the conclusion that the radiation power of an open cylindrical resonator is equivalent to a planar cylinder of radius $a^{eff}$, where $x = a^{eff}/a$ is the root of the equation

$$\ln \left\{ x \frac{(k_0 a)^2}{2} \left[ \pi^2 \left( \frac{2}{\beta} + 1 \right) \right]^{1/4} \right\} = \frac{k_0 a}{2} \left( \frac{1}{\beta^2} - 1 \right)^{1/2} (x - 1). \tag{14}$$

For example, for $\beta = 0.8$ and $\theta = 5°$, we obtain from (14) $a^{eff} = 2.7a$.

This phenomenon can also be described in terms of a change of the impact parameter on which the radiation energy depends exponentially. We compare (9) and (13) for fixed $\beta$, $a$, k, and $\rho_0$. The result can be expressed by saying that if a given radiation power is obtained for a particle passing at a distance h from a closed cylinder, this power can also be obtained if the particle passes at the distance

$$h^{eff} = h + [\beta/[k_0(1-\beta^2)^{1/2}]] \ln [4\beta(2+\beta^2)^{-1/2}/\pi(k_0 a)^2] \tag{15}$$

from a cylindrical resonator tuned to the frequency $k_0$. For example, for $\beta = 0.8$, $\theta = 5°$ $h^{eff} = h + 5.5a$, i.e., the suitable impact parameter increases by nearly three orders of magnitude.

## 4. CALCULATION OF THE CHARACTERISTICS OF THE DIFFRACTION RADIATION ON A COMPUTER

It is not difficult to show that system (5) is of the Fredholm type for arbitrary parameters and, consequently, can be solved by reduction. We note that, although the matrix operator is independent of $\beta$, the order of termination of the infinite series for $B_m$ is determined by the parameter $ka/\beta$, which complicates matters for small $\beta$. This difficulty can be removed by the replacement $\tilde{\mu}_n = \Delta_n \mu_n + i\pi (ka)^2 \times DI_n' [i(1-\gamma)/\beta]^n e^{in\varphi_0}$.

The results of calculation of the frequency dependences of $P_s$ on a computer for various parameters $\beta$, $\theta$, $\varphi_0 = 90°$ are shown in Fig. 1. All quantities are normalized to the magnitude $P_N = 2c\pi\rho_0^2\beta^2 e^{-2qh}$. Above we noted that these curves can be viewed as graphs of the spectral density of diffraction radiation of a single two-dimensional "particle," i.e., of a charged wire. By passing near an open hollow resonator, the wire induces an outburst of radiation with a continuous

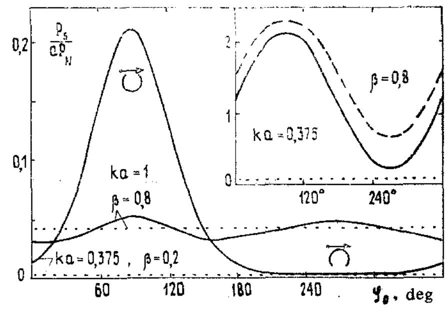


Fig. 4. Radiation energy as a function of the orientation of the coupling slot for $\theta = 5°$. The dashed line corresponds to calculation using formula (13).

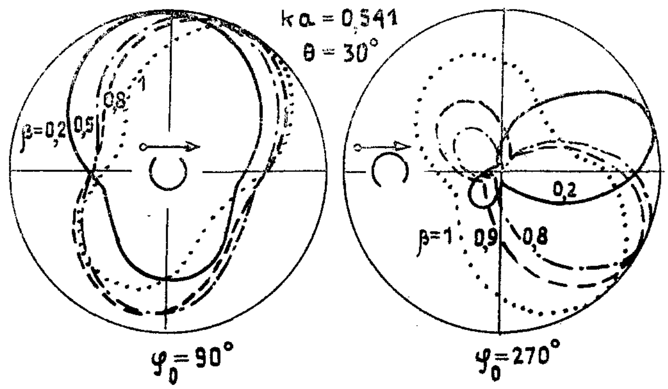


Fig. 5

frequency spectrum, and the energy emitted at the limits of low and very high freqencies tends to zero. This is due to the fact that, for $\omega \to \infty$, the proper field (2) "sticks" to the trajectory of the particle, and weakly perturbs the currents at the surface of the obstacle.

We define an effective width of the radiation spectrum as the frequency band in which the dominant fraction of energy is emitted. An analysis shows that this width is determined by the frequencies of the quasiproper regimes of the cylinder and by the Q-factors (see Fig. 1). The largest contribution comes from the quasistatic resonant regime $H_{00}$ which was studied in [4]. In some cases, one can separate several isolated regions of maximum emission which correspond to oscillations of the type $H_{00}$ and $H_{11}^{\pm}$. Higher types of oscillations manifest themselves weakly. Their contribution, however, is more important in the relativistic case (cf. [7]).

We note that in the motion of charged particles near nonresonant obstacles (strip, closed cylinder), the effective width of the spectrum is determined mainly by the velocity of the particle and by the impact parameter [1]. In the present case, the role of these quantities is considerably less pronounced.

As expected, the resonant oscillations are excited most efficiently when the particle passes above the coupling slot of a resonator ($\varphi_0 = 90°$) (see Fig. 4). This conclusion follows also from formula (13). If the slot is opened in the opposite direction ($\varphi_0 = -90°$), the energy of radiation sharply decreases, which is explained by the fact that the slot enters the region of the "dark pole," i.e., the minimum of the current density function (7). It was noted earlier [2] that a similar phenomenon is observed also for the radiation of a current above a lattice of open cylinders. It is natural to assume that it has an analogous explanation.

Of particular interest is the distribution of the emitted energy in space, i.e., the directionality diagram. As in the case of scattering of a plane wave (see [7]), it is necessary to distinguish clearly the resonant and nonresonant diffraction radiation. When high-Q quasiproper oscillations are excited at the surface of a circular cylinder, a powerful source of secondary radiation appears at the place of the slot, and the directionality diagram therefore discontinuously changes its shape. The directionality diagrams shown in Fig. 5 refer to the case diffraction radiation in the most important case for $k = k_0$, i.e., when the resonant regime $H_{00}$ is excited. They show that if the coupling slot has an optimum orientation ($\varphi_0 = 90°$) the directionality diagram is similar to that for the scattering of a plane wave ($\beta = 1$), and for $\beta \gg 0.2$ it depends weakly on the change of the velocity of the current. If the slot is positioned in the region of the "dark pole," the directionality diagram assumes a specific two-lobe character and is unstable, i.e., it changes strongly with changing $\beta$.

Thus, we have established that the motion of a charged particle near an open resonator is accompanied by the outburst of diffraction radiation with a narrow effective frequency spectrum concentrated around the frequency of the quasistatic resonant regime $H_{00}$. The energy of the radiation is largest in the case when the particle passes above the coupling aperture of the resonator, which reflects the nonuniform character of the proper electromagnetic field of the particle. The directionality diagram of the resonant radiation is stable with respect to the change of velocity of the particle within wide limits. By turning the coupling slot one can control smoothly the energy loss of the particle by radiation.

We note also that the carrier of the field of a nonuniform plane wave of the type (2) can be not only a planar modulated current of charged particles, but also a surface-wave transmission line, e.g., a planar dielectric waveguide [1]. Here the role of the function of the parameter $\beta$ is played by the relative phase velocity $v_{ph}/c$ of the surface wave; the values $\beta \geqslant 0.5$ are most characteristic. In this case, the structure plays the role of a resonant slot antenna excited by the surface-wave transmission line. The above results correspond to the excitation of the characteristics of such an antenna in the approximation of fixed field of the transmission line.

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