Periodicity-Enhanced Plasmon Resonances in the Scattering of Light by Sparse Finite Gratings of Circular Silver Nanowires

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***Abstract*—We consider the H-polarized plane wave scattering by a finite linear grating of circular silver wires using the angular field expansions in local coordinates and addition theorems for cylindrical functions. The study is focused on the influence of the plasmon and the grating resonances on each other. It demonstrates that the scattering per one silver cylinder can be dramatically enhanced if the grating period is tuned to the plasmon-resonance wavelength value.**

***Index Terms*—Grating resonance, light scattering, nanowire grating, plasmon resonance.**

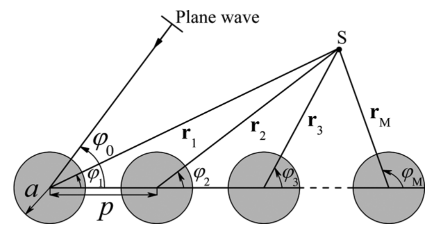

Fig. 1. Scattering geometry and notations.

I. Introduction

As known, material objects can exhibit localized surface-plasmon or quasi-static resonances at certain wavelengths for which the object permittivity is negative [1]–[3]. The physical mechanism behind these resonances is the presence of free electrons in the noble-metal nano-particles that may display collective oscillations giving a major contribution to the dielectric permittivity at optical frequencies. For subwavelength metallic objects, plasmon resonances result in powerful enhancement of scattered and absorbed light that is used in stained glasses and, more up-to-date, in the design of optical antennas and biosensors for advanced applications. Plasmon resonances have unique physical property: in the leading terms, the resonance frequencies depend on the object shape but do not depend on its dimensions [2]. As known, they may shift considerably if the noble-metal particles or wires are collected in closely spaced ensembles, e.g., dimers, trimers, and more complicated configurations [1], [3], [4].

Finite periodic structures made of many particles or wires are even more interesting objects of research as they are strongly wavelength-selective scatterers. As known, periodicity in *infinite* gratings of wires or particles leads to specific phenomena called grating resonances (a.k.a. geometric resonances) [4]–[6]. If the wires diameter $2a$ is a small fraction of the period p, then their frequencies lie just below the frequencies of Rayleigh–Wood "anomalies" (branch points at $\lambda = p/m$, $m = 1, 2, \ldots$ for the normal incidence). The grating resonances are caused by the poles and lead to almost total reflection of the incident plane wave in narrow frequency bands both in the H and E-polarization cases [5], [6]. However it seems that the question of how these resonances display themselves if a grating of wires is *finite* has not been studied so far. Our aim is to analyze their effect on the plasmon resonances of a finite grating made of silver wires.

II. Scattering Problem

Consider a finite grating of M equidistantly located parallel wires illuminated by a plane wave as shown in Fig. 1.

The wires are assumed to be identical infinite circular cylinders, each having radius a and complex relative dielectric permittivity ε. For a 2-D problem, one has to find a scalar function $H_z(\vec{r})$ that is the scattered-field z-component. It must satisfy the Helmholtz equation with corresponding wave numbers inside and outside the cylinders, the tangential field components continuity conditions, the radiation condition, and the condition of the local power finiteness.

The full-wave solution can be obtained similarly to [7], [8], by expanding the field function in terms of the azimuth exponents in the local coordinates (Fig. 1), using addition theorems for cylindrical functions, and applying the boundary conditions on the surface of all M wires. This leads to an infinite $M \times M$ block-type matrix equation where each block is infinite. However we emphasize that, unlike [7], [8], we cast it to the Fredholm second kind matrix equation. It is only in this case that the solution of equation with each block truncated to finite order N converges to exact solution if $N \to \infty$. The results presented

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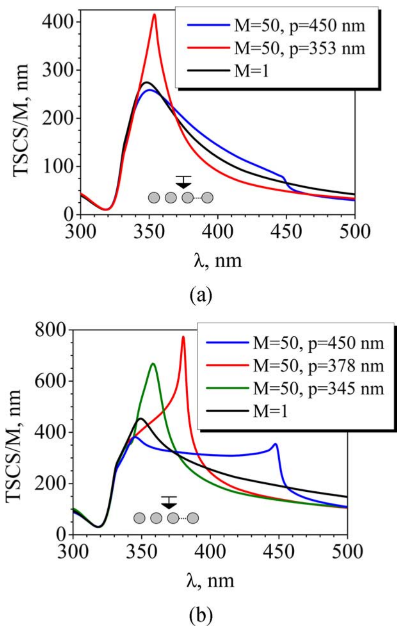

Fig. 2. TSCS per wire as a function of wavelength for the H-wave incident broadside ($\varphi_0 = \pi/2$) on the gratings of $M = 50$ silver nanowires with radii (a) $a = 40$ nm and (b) 60 nm.

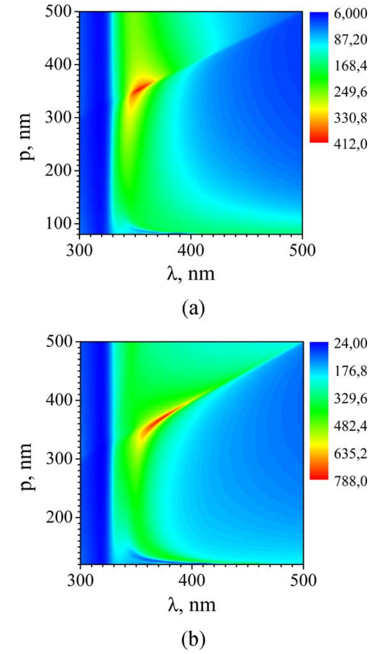

Fig. 3. TSCS per wire as a function of wavelength λ and period p for the H-wave incident broadside ($\varphi_0 = \pi/2$) on the gratings of $M = 50$ silver nanowires with radii (a) $a = 40$ nm and (b) 60 nm.

below were computed with $N = 4$; this provided 3 correct digits in the far-field characteristics of the gratings of silver wires with radii $a \leq 75$ nm and periods $p \geq 200$ nm. Note that denser gratings need larger values of N to achieve the same accuracy.

III. Results and Discussion

In view of limited space we restrict numerical results to the scattering of the H-polarized plane waves by sparse ($\rho - 2a > a$) finite gratings of subwavelength silver nanowires. The results for the dense gratings will be presented in a fuller publication. In Figs. 2–5, the wavelength varies from 300 nm to 500 nm and the complex-valued dielectric function of silver has been borrowed from [9].

The most interesting question is what the peak values of the grid characteristics become if the plasmon and the grating resonance wavelengths coincide. Here, the main instrument of tuning them together is the period of the wire grating, because the plasmon resonance has almost fixed wavelength near to the single-wire value around 350 nm. For example, in Fig. 2(a) and (b) presented are the plots of the total scattering cross-section (TSCS) as a function of the wavelength for stand-alone silver nanowires with radii 40 nm and 60 nm. They have broad maxima at 348 nm and 349 nm, respectively.

In Fig. 2, presented are also the plots of TSCS per wire as a function of the wavelength for several sparse gratings of $M = 50$ silver wires with radii 40 nm and 60 nm. One can see that if the period is far from the plasmon resonance wavelength, the grating resonance has almost no effect on TSCS: it shows up as a small spike standing on the slope of plasmon resonance. This behavior changes drastically, however, if two resonances are tuned together – in that case there is one much more intensive resonance. As visible, enhancement in TSCS can reach 1.51 and 1.7 for a grating of $M = 50$ wires with $a = 40$ nm, $p = 353$ nm and $a = 60$ nm, $p = 378$ nm, respectively. The combined peaks are red-shifted relatively to plasmon resonance of a single wire. As we have found, the optimal wire radii to observe the strongest TSCS enhancement lie in the range of 25 nm to 70 nm.

In Fig. 3, presented are reliefs of per-wire TSCS (i.e., normalized by M) as a function of two parameters: wavelength and period, for gratings of 50 silver wires with radii 40 nm and 60 nm. In each case one can see a sharp "ridge" stretching along the line $\lambda = p$ that marks the grating resonance. Another, broader ridge stands at the fixed wavelength near to 350 nm – this is plasmon resonance. The areas of strongly enhanced TSCS are located at the junctions of these two ridges.

In Fig. 4, presented are the near-field amplitude and phase patterns for the 10-wire sparse grating with $a = 40$ nm at the wavelength of the combined resonance. The field hot-spot maxima are visible near the illuminated sides of the wires similarly to the single-wire case [1]. However besides of that one can clearly see two local standing waves near to the grating: one is above the illuminated side – it is formed by the incident plane wave and the reflected field. Another is an "orthogonal" wave standing along the wires; it is formed by two oppositely propagating $\pm$1st quasi-Floquet harmonics of the grating as the wavelength is just above the value $\lambda = p$.

Although a comparison between infinite and finite gratings is very interesting, it is not so easy to select a common scattering characteristic. As such a quantity, we propose the *reflective efficiency*. For a finite grating this is the part of TSCS linked to the light scattered into the upper half-space and normalized by $2aM$, $R_{eff}^{(M)} = 1/(\pi M ka) \int_0^\pi |\Phi(\varphi)|^2 d\varphi$, where the far-field scattering pattern is defined as $\Phi(\varphi) = \lim_{r\to\infty} H_z(\vec{r})\sqrt{i\pi kr/2}e^{-ikr}$. For infinite grating, this quantity is the power reflectivity [10] normalized by $2a/p$. In Fig. 5, presented is such comparison for several gratings of silver

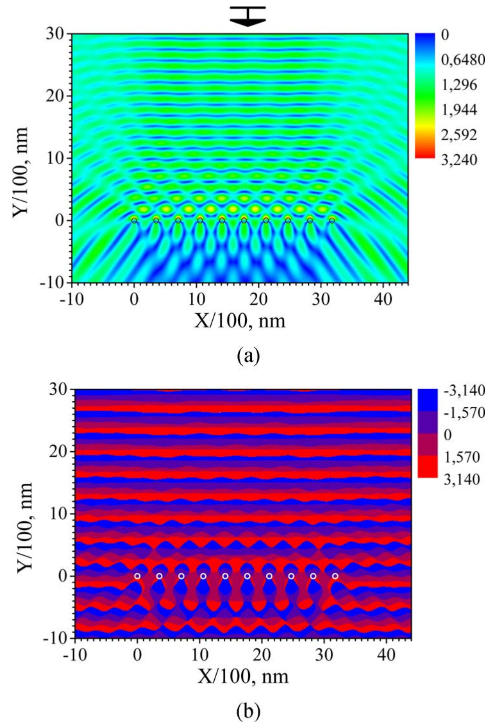

Fig. 4. (a) Near-field amplitude and (b) phase patterns for the H-wave incident normally ($\varphi_0 = \pi/2$) at the grating of $M = 10$ silver nanowires with radii $a = 40$ nm, period $p = 353$ nm, in combined resonance at $\lambda = 353.1$ nm.

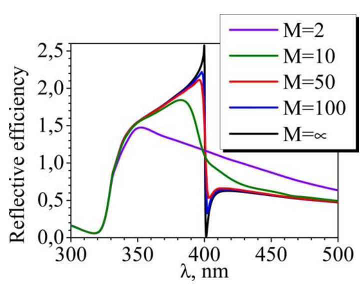

Fig. 5. Normalized reflective efficiency as a function of wavelength for finite and infinite gratings of silver nanowires with the radii of 60 nm and period 400 nm. The H-pol plane wave is at normal incidence.

nanowires with $a = 60$ nm and $p = 400$ nm. As visible, a chain of 10 wires of this radius provides reflective efficiency within 10% of the infinite grating value in the whole visible range except for the narrow band around the grating-type resonance where at least 100 wires are needed to achieve the same effect. Note that below the plasmon resonance and above the grating resonance the reflective efficiency is close to its value for one silver wire. This is what can be expected for a sparse grating.

IV. Conclusion

If an H-polarized plane wave is incident normally at a sparse finite linear grating of silver nanowires, plasmon and grating-type resonances are observed as two separate peaks in the per-wire scattering cross-sections; when tuned together they enhance each other. The optimal for such enhancement radius of wire is between 25 and 70 nm, and the sharpness of combined resonance peak depends on the number of wires. Around 100 wires provide the per-wire reflective efficiency within 90% of infinite grating including in the grating resonances. An interesting observation is a specific double standing wave near to the grating in the grating resonance.

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References

[1] J. P. Kottman and O. J. F. Martin, "Plasmon resonant coupling in metallic nanowires," *Opt. Express*, vol. 8, pp. 655–663, 2001.

[2] D. R. Fredkin and I. Mayergoyz, "Resonant behavior of dielectric objects (electrostatic resonances)," *Phys. Rev. Lett.*, vol. 91, pp. 3902–3905, 2003.

[3] W. Ding, R. Bachelot, R. Espiau de Lamaestre, D. Macias, A.-L. Baudrion, and P. Royer, "Understanding near/far-field engineering of optical dimer antennas through geometry modification," *Opt. Express*, vol. 17, no. 23, pp. 21228–21239, 2009.

[4] F. J. G. Garcia de Abajo, "Colloquium: Light scattering by particle and hole arrays," *Rev. Mod. Phys.*, vol. 79, no. 4, pp. 1267–1289, 2007.

[5] R. Gomez-Medina, M. Laroche, and J. J. Saenz, "Extraordinary optical reflection from sub-wavelength cylinder arrays," *Opt. Express*, vol. 14, no. 9, pp. 3730–3737, 2006.

[6] V. O. Byelobrov, J. Ctyroky, T. M. Benson, R. Sauleau, A. Altintas, and A. I. Nosich, "Low-threshold lasing eigenmodes of an infinite periodic chain of quantum wires," *Opt. Lett.*, vol. 35, no. 21, pp. 3634–3636, 2010.

[7] V. Twersky, "On a multiple scattering theory of the finite grating and the Wood anomalies," *J. Appl. Phys.*, vol. 23, no. 10, pp. 1099–1118, 1952.

[8] H. A. Ragheb and M. Hamid, "Scattering by N parallel conducting circular cylinders," *Int. J. Electron.*, vol. 59, pp. 407–421, 1985.

[9] P. B. Johnson and R. W. Christy, "Optical constants of the noble metals," *Phys. Rev. B*, vol. 6, pp. 4370–4378, 1972.

[10] V. O. Byelobrov, T. M. Benson, J. Ctyroky, R. Sauleau, and A. I. Nosich, "Plasmon and structure resonances in the scattering of light by a periodic chain of silver nanowires," in *Proc. Int. Conf. Transparent Optical Networks (ICTON-2010)*, Munich, 2010, Paper Tu.P.8.