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Trends in microdisk laser research and linear optical modelling

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Abstract Research into microdisk lasers demonstrates new achievements both in the technology and in the associated physical effects and applications. Melting and rounding of the disk edge boosts the Q-factors due to improved surface smoothness. In-plane cavity shape is widely used as a design instrument. Optimal shaping of pumped area lowers the threshold power. Photonic molecules made of several microdisks as "photonic atoms" show lasing at several closely spaced frequencies. A microdisk with a single quantum dot as an active region is considered as the most promising system for realisation of a single photon emitter necessary for quantum computing. These new effects and devices can be simulated with accurate numerical techniques, developed recently for "warm-cavity" linear modelling, that are able to bring a new vision of the physics of lasing.

Keywords Microcavity laser · Active region · Single quantum dot · Photonic molecule · Q-factor · Linear threshold · Integral equations

1 Introduction

Although disk-like shapes had been attracting the laser scientists since the 1970s, semiconductor lasers in the form of thin disks on pedestals were first successfully demonstrated in the early 1990s as extremely compact sources of light (McCall et al. 1992; Hovinen et al. 1993; Mohideen et al. 1994). These disks, usually 1–10 μm in diameter and 100–300 nm in

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thickness, were made of GaAs and contained one 5–10 nm thick GaAs/AlGaAs or GaAs/InP quantum well (QW) as an active region to provide the emission wavelength λ around 1550 nm. The main features of such lasers are (i) ultralow thresholds, (ii) periodically spaced frequencies of lasing within the QW photoluminescence band, and (iii) predominantly in-plane light emission. The lasing modes are identified as whispering–gallery (WG) modes confined at the rim due to nearly total internal reflection (Matsko and Ilchenko 2006). Through the past decade, the sophistication of such lasers has followed several routes closely tied to the progress in manufacturing technologies including lithography, wet and dry etching, epitaxial growth, and thin-film deposition. First of all, the single QW as an active region was upgraded to cascaded multiple-QW structures (Corbett et al. 1996; Faist et al. 1996; Zeng et al. 1999). Then QWs were replaced with layers of randomly grown quantum boxes (Gayral et al. 1999) and eventually quantum dots (QDs) (Cao et al. 2000; Michler et al. 2000). A QD-based active region has advantage over a QW one, offering less dependence on temperature and a narrower photoluminescence band. Both factors are important for achieving single-mode laser operation. New active-material systems have been also tried, to cover wavelength bands from infrared to ultraviolet (Bagnall et al. 2000; Zwiller et al. 2003a; Liu et al. 2004): GaInP/InP ($\lambda = 650$ nm), ZnSe/CdS (510 nm), ZnO/SiO_2 (390 nm), and InGaN/GaN (370 nm). On the other hand, cascading of the 20–30 periods of three-QW active regions has enabled the researchers to design microcavity lasers with the emission wavelengths as long as $\lambda = 5\,\mu$m (Faist et al. 1996), $\lambda = 10\,\mu$m (Gianordoli et al. 2000) and longer (Fasching et al. 2007). As a rule, the first experiments with new materials were performed with optical pumping and at cryogenic temperatures. On refining the technologies, the next step was usually to achieve continuous wave room temperature lasing and demonstrate a device with current injection (Baba et al. 1997; Fujita et al. 1999, 2000; Zhang and Hu 2003; Ide et al. 2005). Injection lasers, however, immediately revealed that the proper placement of electrodes plays a crucial role in their performance.

Today's trends in microcavity lasers research are associated with smoothing of the cavity rim, optimising the shape of the pumped area, looking for optimal shapes of cavities, integrating microdisks with optical fibres and annular Bragg-like reflectors, building arrays of microdisks, and shrinking the active region to a few individual QDs.

For comprehensive modelling of the microcavity laser, one has to account for several fundamental mechanisms such as transport of carriers, stimulated emission of photons, heating, and optical field confinement. Nonlinear effects link them together and limit the output power (Streiff et al. 2003; Piprek 2005). However, a reasonable reduction of complexity can be achieved within a linear optical problem neglecting all non-electromagnetic phenomena and viewing the optical modes of a cold cavity as solutions to Maxwell equations with conditions of the field tangential components continuity across the cavity boundary and a radiation condition at infinity.

Traditionally, cold-cavity linear modelling of microdisk lasers neglects the pedestal, which is assumed to have no effect on WG modes, and nonuniformity in the z direction (the z-axis being the disk axis), due to the very small thickness of QW layers. It also uses a reduction of dimensionality from 3-D to 2-D in the disk plane (see Fig. 1), with the effective-index method (Marcuse 1982; Frateschi and Levi 1996). Further, the goal is finding the frequencies and Q-factors of the natural modes of the *passive cavities*. The methods applied range from the analytical WKB technique (Frateschi and Levi 1996) and the ray-tracing-based billiard theory (Noeckel and Stone 1996; Gmachl et al. 1998; Schwefel et al. 2004a,b) to the allnumerical FDTD approximations (Li and Liu 1996; Hagness et al. 1997; Fujita and Baba 2001; Huang et al. 2004; Yang et al. 2007). Recent progress has been made in two directions. The first is systematic use of the boundary integral equations to study the passive-cavity modes

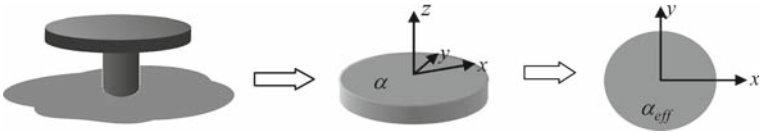


Fig. 1 Reduction of the layered disk-on-pedestal electromagnetic problem to the homogeneous 3-D free-space disk and to the 2-D circular-cavity model based on the effective refractive index concept

more accurately; the second is development of a new model that macroscopically accounts for the presence of *active region* and enables one to extract the lasing thresholds from the electromagnetic field equations.

We present our view of these trends below. Note that we restrict ourselves to the electromagnetic-theory modelling based on the linear time-harmonic Maxwell equations. This means that all aspects of the quantum confinement, associated QW and QD differences, and other quantum-mechanical effects are out of the scope of this review.

2 Smoothing and shaping of the disk rim

All real disks have finite thickness and are not perfectly smooth. To understand the role of surface roughness, it is useful to review first the basic properties of the WG modes in 2-D, i.e. in ideally smooth cylinder of radius a and refractive index α. Here all modes except the monopole mode are twice degenerate, i.e. two modes have the same frequencies and thresholds. The corresponding modal fields are orthogonal: they are standing waves, one having the field dependence on azimuth as $\cos m\varphi$ and the other as $\sin m\varphi$, where $m = 1, 2, \ldots$ (The frequently held belief that there exist "clockwise" and "counter-clockwise" WG modes $\sim e^{\pm im\varphi}$ is wrong both mathematically and physically; this belief is caused by the fact that, because of the degeneracy, any linear combination of the standing waves mentioned is also a natural mode.) The standing-wave modes display WG-type behaviour when $m \gg 2\pi a/\lambda \gg m/\alpha$ (Frateschi and Levi 1996; Smotrova et al. 2005a). Both the natural frequencies of a passive disk and the lasing frequencies of a disk with gain can be well approximated by the formula $\lambda \approx 2\alpha a/(\pi m)$, which follows for $2\pi\alpha a/\lambda \gg m \gg 1$ from the equation obtained by neglecting the radiation losses, $J_m(2\pi\alpha a/\lambda) \approx 0$. The corresponding "free spectral range" (i.e., the distance between the frequencies of two lasing modes) is found as $\Delta f = \pi c/(2\alpha a)$, where c is the velocity of light. The WG modes' radiation losses (i.e., the values of $1/Q$) and their thresholds behave as $e^{-const\ a/\lambda}$. Therefore in an ideal 2-D circular cavity the larger the radius a or the azimuth modal index m the higher the Q-factor.

Finite thickness of microdisk does not lift the degeneracy of the WG-mode doublets; however it limits their accessible Q-factors by a certain (still very high) value, which depends on the thickness-to-radius ratio.

The roughness of the microdisk rim has a strong effect on lasing in two respects. The first is that if the contour is perturbed from a circle, each standing-wave mode obtains a specific shift in frequency. Thus, two modes split and make a doublet. In the first experiments, the surface roughness of microdisks was quite high; however, the spectral resolution of receivers was too low and thus WG-mode doublets were usually not resolved. They became clearly visible in the spectral curves only when the disk roughness size exceeded 20 nm (see Fig. 4 of Gayral et al. 1999). In contrast, recently fabricated high-finesse silicon disks (radius 5–30 μm, roughness about 2 nm, $Q = 5 \times 10^6$) demonstrated splitting of the WG mode doublets by less than 1 picometre, as measured with an extremely high-resolution spectrometer (Borcelli et al. 2005). In fact, the existence of close doublets undermines the concept of free spectral range being inversely proportional to the disk radius—this is true only in the case of an ideal circle.

The second effect is the high diffraction losses of each mode of the doublet if the scale of roughness becomes comparable to the wavelength in the disk material. This drastically limits the growth of the Q-factors of passive disks (or the reduction of thresholds in active disks) with greater frequency or disk radius or azimuthal index of a WG mode to the limit (usually very high) set by the finite disk thickness. Until recently, measurements of microdisk Q-factors have usually shown values in the range 10^3–10^4 (Vahala 2003). This is much lower than the Q-factors of 10^8–10^9 registered for the WG modes in spherical microcavities fabricated by melting a tip of a glass fibre (Gorodetsky et al. 1996; Vahala 2003), where the surface tension forces make the sphere "atomically smooth" by reducing the roughness to 1–2 nm level. Such a disadvantage has prevented disk cavities from use in many important applications. Therefore several technologies for selectively melting the rim of the disk resonator have been developed (Armani et al. 2003; Kippenberg et al. 2003, 2004; Spillane et al. 2005; Kalkman et al. 2006) and immediately showed record-high Q-factors, 10^7–10^8. These resonators are called "toroidal" however in fact they are closer to the disk ones with rounded or rolled-up rims (Fig. 2a). Indeed, they have no holes in the centre and their rim rounding radius is just a couple of wavelengths—thus there is no WG effect in the cross-section. Still this combined effect of smoothing and rounding the rim seems to be a major success. Recently such cavities doped with erbium have been used as ultra-low-threshold lasers (Kalkman et al. 2006) and their importance for the Cavity Quantum Electrodynamics has been emphasized (Spillane et al. 2005).

On the other experiments, truly toroidal passive crystalline cavities were fabricated by (Ilchenko et al. 2001) and showed Q-factors near to 10^7. Another fine technology based on a

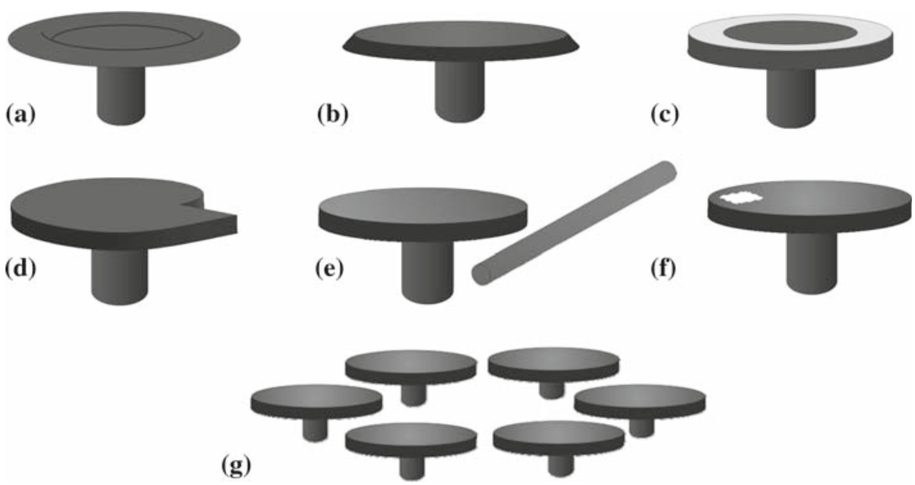


Fig. 2 Schematic images of microdisks mentioned in the paper: (**a**) disk with a rounded rim, (**b**) disk with a wedge-shape rim, (**c**) disk with a ring-shape active region, (**d**) disk having "spiral"-shape cavity, (**e**) disk loaded with a tapered optical fibre, (**f**) disk with single quantum-dot active region, and (**g**) cyclic photonic molecule of disks. Note that, besides of the shown here air-clad on-pedestal versions, these structures can be also manufactured as disk-on-substrate ones. This generally leads to lower Q-factors due to smaller refractive index contrast, but provides better mechanical and thermal properties

wedge-shape etching of the disk rim (Fig. 2b) has led to Q $\sim 10^6$ values (Polman et al. 2004). Interestingly, the authors of (Ide et al. 2005) claim that it was the change of the sidewall to the disk face inclination, from 50° to 80°, which enabled them to double a microdisk Q-factor.

3 Active region shape variation

Until recently, experiments with optically pumped microdisk lasers used wide, e.g. 20–50 μm in diameter, pump-laser beams, to provide uniform illumination of the upper faces of microdisks. However, as any WG mode has the field concentrated in a narrow ring-shape area around the disk rim, it is obviously enough to illuminate only this area (Fig. 2c). The task of shaping the pump beam into a hollow one, with zero intensity in the centre, was elegantly solved in (Rex et al. 2001) by using an axicon, and a reduction of pump power was clearly observed. Similar understanding of the importance of matching the shape of the active region with WG mode field pattern has led to the design of injection lasers with ring contacts (Kneissl et al. 2004; Ben-Massaoud and Zyss 2005).

Keeping this in mind, one may realize that injection lasers with electrodes placed in the centre of circular disk, like those in Fig. 1 of Fujita et al. (1999), have much higher thresholds of the WG modes than the ring-electrode lasers. On the other hand, when the injection contact was placed in the centre of a stadium-shape microcavity, it was found that a powerful lasing could occur on the "bow-tie" modes, of non-WG character (Gmachl et al. 1998; Gianordoli et al. 2000). Today it is widely recognised that proper positioning of the electrical contacts is the most important consideration in the design of all injection microcavity lasers (Bhattacharya et al. 2005).

4 Cavity shape variation

A serious drawback of microdisk lasers is the low directionality of light emission inherent in circular cavities. As mentioned, WG modes in an ideal disk behave as $\cos m\varphi$ or $\sin m\varphi$ at any distance from the disk centre, thus the far-field power emission patterns display $2m$ identical beams. Therefore it is clear that improvement of the directionality needs a distortion of the cavity shape from the circle. The first experimental attempt of this kind was reported in Levi et al. (1993) where a microdisk laser with a hump on the rim was measured. The next great step ahead was the discovery of the so-called "bow-tie" modes in the stadium-shape dielectric cavity, and the realization of a semiconductor microlaser based on such a cavity (Gmachl et al. 1998). This device showed lasing into four intensive emission beams, because of the two-fold symmetry, instead of several tens as typical for a microdisk, however with much higher thresholds. Ling et al. (2003) reported similar properties for a deformed square ring hybrid glass microlaser, and Kim et al. (2004b), measured directive emission from elliptical microdisks. Quadrupolar and hexadecapole shapes were tested in Gianordoli et al. (2000) and Schwefel et al. (2004a) with pretty similar conclusions. Large-size diamond-shaped 2-D cavities with airgaps were reported by Fukushima et al. (2007). On the other hand, reduction of the threshold was predicted (Fujita and Baba 2001) and then demonstrated (Fujita and Baba 2002) for a thin gear-shape GaAs/InP cavity in air on a pedestal. This effect is observed provided that the periodicity of indentations on the disk rim is matched with the working WG mode azimuth index. However, such a deformation of the cavity shape has no effect on the emission directionality because the microgear laser displays the same $2m$ beams as a simple disk. It should be noted that the dependence of the Q-factors and far-field patterns on the cavity shape has attracted remarkable attention from theoreticians (see below) who have studied a variety of shapes. However, the tradeoffs between directionality and threshold have not been studied systematically yet.

A real breakthrough in terms of directionality was the “spiral” microdisk laser (Fig. 2d), whose rim follows an Archimedes spiral with endpoints joined through a small step in radius, around $0.1a$ wide. Such devices were first manufactured as InGaN pillars with photo-pump (Chern et al. 2003) and later as thin disks on a substrate with carrier injection (Kneissl et al. 2004). They also fully exploited the advantages of the ring-shape pumping. For the first time, they demonstrated a sort of “unidirectional” emission with a single 40°–60°-wide main beam. This was achieved at the expense of a much higher threshold as the step on the disk rim strongly perturbs the high-intensity domain of the WG mode field. Recently this idea was reproduced in Ben-Massaoud and Zyss (2005) with a polymer disk.

Characterisation of emission directionality is, in fact, a study of the laser as an active optical antenna. Therefore it is convenient to borrow some basic principles and quantities from microwave antenna theory and design. For instance, directionality can be conveniently quantified with the aid of *directivity* as a ratio of power emitted into the main beam direction to the total power averaged over all possible directions (Smotrova et al. 2006a; Boriskina et al. 2006; Dettmann et al. 2008). Then it is easy to see that directivity is 1 for the omnidirectional emission of the monopole mode ($m = 0$) and 2 for any other ($m = 1, 2, \ldots$) mode of a circular cavity. If a small roughness is present, then the directivity differs from 2 but stays close to this value. For comparison, both the computations made for the disks with point-like perturbations and experimental far-field emission patterns of the spiral microcavity lasers mentioned above show directivity values around 10.

5 Annual-Bragg-reflector assisted microdisks

Besides of the almost total internal reflection, another famous mechanism of the light confinement is the Bragg effect due to reflection from a periodic stack of pairs of dielectric layers. Therefore the idea of placing a microdisk inside an annular Bragg reflector (ABR) attracted the early attention of developers (Labilloy et al. 1998). In the 2000s several groups have been researching into the fabrication and characterization of such lasers both in infra-red and ultra-violet bands (Jebali et al. 2004; Scheuer et al. 2004, 2005a,b). As ABR-assisted microdisks are usually etched on the surface of a substrate, it is easier to collect the emitted light in the normal to surface direction (Scheuer et al. 2005a,b). Still a design with output waveguides penetrating the ABR is also studied (Jebali et al. 2006). The number of ABR pairs varies from units to over a hundred. Basically it depends on the contrast between the low and high-index layers, in terms of effective refractive indices, that means on the depth of the ABR grooves. An important observation is that when using ABRs the grating pair nearest to the microdisk layers should have radii deviating from the quarter-wavelength rule borrowed from planar Bragg reflectors (Scheuer and Yariv 2003).

6 Coupling of microdisk to optical fibre

The directionality of light emission from microdisk laser can be radically improved, without distorting the circular shape responsible for ultralow thresholds, by using a completely different approach. This is to load a microlaser cavity with an optical fibre (Fig. 2e). The fibre should, however, be tapered from a conventional diameter of around a hundred microns to at least several microns. Here, the field structure of the disk WG modes suggests that the fibre should be located near to or in the same plane as the disk. Furthermore, the separation between the disk rim and the fibre should not be too small otherwise the WG mode passive cavity Q-factor or active cavity threshold would be severely spoiled.

Various combinations of microdisk and microring lasers with optical fibres and rib optical waveguides have been designed (Hagness et al. 1997; Zhang et al. 1996). Borcelli et al. (2004) presented an elegant experimental technique for a high-precision evaluation of Q-factors of passive microdisks (pump switched off) by measuring the light transmission through an undercoupled fibre at a wavelength slightly shifted from the lasing one. Then the same setup was used to register the light emission at the lasing frequency when the pump was applied. On the other hand, a vertical coupling configuration that features a microdisk or an array of microdisks located on top of the buried optical waveguide has been successfully demonstrated (Tishinin 1999; Little et al. 1999; Klunder et al. 2001; Djordjev et al. 2002; Choi et al. 2004).

7 Single quantum dot microdisk lasers

The interest in single-QD lasers is caused by the hope to realise a source of single photons necessary for quantum computing (Zwiller et al. 2004). First estimations for a QD located externally to the surface of a high-Q microsphere (Pelton and Yamamoto 1999; Gotzinger et al. 2001) promised remarkable features for such lasers. However, it was soon realised that individual QDs could not be easily placed on the surface of microsphere in a controlled manner. This is in contrast to a layer of QDs that is obtained by simply immersing a sphere into a solution containing the QDs (Yang and Vahala 2003; Shopova et al. 2004). Therefore the attention of experimentalists has switched to microdisks and microposts (Gerard and Gayral 2001). Today preference is given to the former cavities as they have much greater Q-factors. The lasing of a microdisk with a single QD (Fig. 2f) having size 30–50 nm and height of 5–10 nm was reported in Zwiller et al. (2003b). Here, an excellent technology was proposed: first individual QDs were grown on a substrate, then gold markers were deposited to map the QDs, and finally etching was used to cut out a disk on a pedestal, with the desired location of a selected QD. Another emerging technology is based on optical-force manipulation of QDs (Lester et al. 2001).

8 Photonic molecule lasers

Photonic molecules (PMs) are finite arrays or aggregates of closely spaced and optically coupled microcavities with each microcavity viewed as a “photonic atom” (Fig. 2g). They display a splitting of each mode of an individual resonator to bonding and anti-bonding “supermodes” of different types, the total number of supermodes being the same as the number of resonators times the degeneracy order of an isolated-cavity mode (Evans and Holonyak 1996; Bayer et al. 1998). In the special case of circular disk cavities, where each unperturbed mode having $m \neq 0$ is double degenerate, it equals twice the number of disks. If all or some of such optically coupled resonators contain active regions, they may start lasing provided that the overall gain is above threshold. The first PM lasing was experimentally realised with circular arrays of two, four and six microrings on a substrate (Evans and Holonyak 1996). The authors anticipated a reduction of threshold relatively to the single disk, caused by the symmetry-assisted coupling between the cavities, however instead measured a twice higher threshold value. Recently, microlasers based on twin disks, 3×3 matrix and linear-array PMs of disks on pedestals were demonstrated by Nakagawa et al. (2005). The main attention was paid to the formation of multiplets of lasing frequencies.

9 Modelling: from billiard theory and FDTD to integral equations

As mentioned, today's etching and epitaxy-based technologies are well developed and enable controlled fabrication of thin cavities. 3-D optical-field problem for a thinner-than-wavelength disk can be approximately reduced to the 2-D one, in the disk plane, with the effective-index approach (Marcuse 1982; Frateschi and Levi 1996; Smotrova et al. 2005a). This approximation agrees well with the predominance of the in-plane radiation and leads to the separate analysis of the E_z and H_z-polarized modes, with a conclusion that the latter are dominant. One aspect here frequently escapes the attention of researchers: effective index is not a uniquely introduced quantity but takes discrete values depending on the mode of the infinite slab that approximates the z-dependence of the full field.

In 2-D, the simplest shape is circle—it can support 2-D WG modes and can be studied analytically (Frateschi and Levi 1996). Even such simple shape is not in all aspects an easy object for modal analysis—e.g., a good approximation to explain the measured lasing frequencies is obtained if one assumes that the electric field vanishes at the disk rim. However even rough estimation of the WG-mode Q-factors needs a much finer technique such as the tunnelling considerations or Debye asymptotics for cylindrical functions. More recently, this analysis has been extended to circular cavity with an asymptotically small linear defect (Apalkov and Raikh 2004). This analysis revealed directional scattering of the modal field by the defect, although total radiation pattern and removal of degeneracy were not assessed there. These aspects were studied in Dettmann et al. (2008) for the similar problem of a point-like defect in circular cavity. More complicated separable 2-D configurations are concentric circular ones, which have important prototype—ABR-assisted disk-on-substrate lasers. Modelling of such passive cavities has been attempted many times in the 2000s (Kaliteevski et al. 2000; Scheuer and Yariv 2003; Jebali et al. 2004, 2007), although results published contain only the information on the frequencies and not Q-factors; the analysis of Jebali et al. (2007) appears inadequate as it neglected the mode discreteness and used different field functions for estimating the stored and the radiated power values.

The need for more directional emission than $2m$ identical beams of ideally circular disk has stimulated research into the modelling of 2-D cavities of various other shapes. The most important role in the 1990s was played by the ray-tracing “billiards theory” (Noeckel and Stone 1996; Gmachl et al. 1998; Noeckel and Chang 2002; Schwefel et al. 2004a,b). One of the achievements of this theory was the discovery of so-called “bow-tie” modes in the stadium cavities. The billiards theory is able to predict the mode frequency; however, being essentially a geometrical-optics approach it fails to accurately characterize the Q-factors and emission directionalities. An attempt to improve it by adding Fresnel coefficients has limited effect as based on the assumption of a locally flat boundary illuminated by a locally plane wave that is far from reality for 5–20 λ microcavities. More generally, billiards theory fails to grasp the discreteness of natural modes in terms of complex frequencies. The same relates to a more accurate modification of this theory in the form of “Gaussian-optical approach” of (Tureci et al. 2002) that is based on the parabolic-equation approximation.

Vector-mode 3-D modelling of microcavities has been attempted so far only with commercial and home-made finite-element and FDTD codes (Li and Liu 1996; Hagness et al. 1997; Fujita and Baba 2001; Huang et al. 2004; Campenhout et al. 2005; Spillane et al. 2005;

Yang et al. 2007). Note, however, that the FDTD and other time-domain-based numerical codes that are so popular today are not able to solve eigenvalue problems directly. Instead, they need a pulsed source placed inside a cavity, so that evaluation of the natural frequencies and Q-factors is done via studying a transient signal. This approach is therefore strongly dependent on the choice of the source and observation points and generally cannot provide controlled accuracy, especially if $Q > 10^4$.

Therefore it is not surprising that the recent progress in the linear optical modelling of microlaser cavities is associated mainly with the shift from rough analytical and numerical estimations to accurate full-wave integral-equation (IE) analyses. So far this has concerned only 2-D models of thin cavities.

There are two alternative formulations of IE: (a) volume IE (VIE) and (b) boundary IE (BIE). The first has the advantage of being applicable to cavities with both constant and spatially varying refractive indices (Kim et al. 2004a). However, 3-D VIE and also 2-D VIE in the most interesting case of the H-polarisation are strongly singular, and this makes their application questionable because of the non-convergence of discrete schemes. This drawback was overcome in Kottmann and Martin (2000), where a regularisation procedure based on the inversion of the circular-cell parts in the meshed 2-D VIEs was developed. In contrast, the E-polarised 2-D case VIE is a well-posed Fredholm second kind equation with logarithmic singularity and results in convergent algorithms (Zhuck and Yarovoy 1994). A VIE method in combination with the perturbation technique has always been a traditional tool for studying surface roughness effects. Applied to the microdisk analysis in Borcelli et al. (2004, 2005) and Srinivasan et al. (2005), it has enabled analytical estimation of the removal of WG-mode degeneracy, i.e. the frequency splitting and the Q-factor spoiling. In the 2-D theory of light filtering by a microdisk cavity filter, a VIE technique was developed in Chremmos and Uzunoglu (2004a,b). This approach has clear advantages over such popular counterparts as the physically transparent, yet rough, coupled-mode approximation (Hammer 2002; Zhang and Grischkowsky 2003; Morand et al. 2004) and FDTD codes (Hagness et al. 1997). The same VIEs, if extended into the complex-frequency domain, can be used for accurate calculation of the Q-factors.

BIE formulations are more economic than VIE ones as they have lower dimensionality. Although they work only if the refractive index is constant inside the cavity, BIEs can be easily cast into the form free of strong singularities. A good proof of their power can be seen in the recent study revealing the true nature of the "bow-tie" modes in the stadium cavity (Wiersig 2006). However, BIEs may suffer from another very serious drawback. Many forms of BIE possess an infinite number of discrete defect frequencies (Wilton 1992)—eigenvalues of the interior electromagnetic problem where the boundary is assumed perfectly electrically conducting and inside filling has the material parameters of the outer medium (e.g., free space). In terms of an eigenvalue problem, it means that an infinite number of false real-valued eigenfrequencies are present simultaneously with true complex-valued ones. Such a "defective" BIE technique was developed in Wiersig (2003a). It was applied to the analysis of passive hexagonal cavities in Wiersig (2003b); Nobis and Grundmann (2005) and stadium and rounded-triangular cavities in Lee et al. (2004); Wiersig (2006) and Kurdoglyan et al. (2004).

It is therefore not surprising that only low and medium-Q modes could be studied (in Wiersig (2003b), it was even stated, erroneously, that high-Q modes are of no interest for laser applications). Another version of "defective" BIE was applied in Rogobete and Henkel (2004) to study the enhancement of spontaneous emission from variously shaped cavities in a more justified manner—only smaller than wavelength cavities were computed, i.e. the frequency remained lower than the first "defect" value.

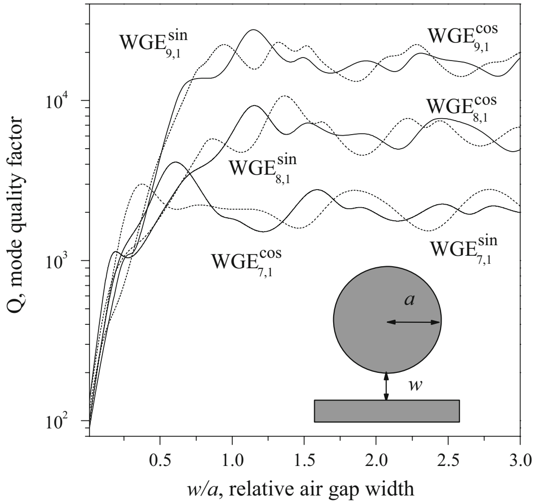


Fig. 3 Q-factors of the $(E_z)_{m,1}$ modes in a circular cavity coupled to a slab waveguide versus the airgap width normalised to the disk radius; both refractive indices are $\alpha_{\rm eff} = 2.64$; slab thickness is $0.5a$

It should be noted, however, that refinement of the discretisation scheme with an analytical regularisation technique may reduce the negative effect of false eigenvalues (although it does not remove it), so that rather high-Q modes can be computed. Such a refined variant of "defective" 2-D BIE analysis has been successfully applied in Boriskina et al. (2002) and Boriskina et al. (2003) to systematic study of the WG modes in circular-ring and elliptic cavities in layered media (see Fig. 3).

Mathematically, the most reliable tool for a modal analysis of dielectric cavities should use the so-called Muller BIE (in fact, two coupled BIEs in 2-D or four in 3-D) because they (i) are free of defect frequencies and (ii) have smooth or integrable kernels, and (iii) are of the second kind. The Muller BIE can be discretised either with collocations (Rokhlin 1990) (i.e., meshing the boundary and introducing local basis functions) or with a Galerkin-type projection to global expansion functions (Boriskina et al. 2004a). As the laser cavity commonly has a convex or at least star-like boundary, the latter way of discretisation leads to a more economic algorithm, although both ways possess a convergence thanks to the Fredholm second kind nature of Muller BIEs. According to Boriskina et al. (2004a), the size of the resultant matrix is determined by the optical size of the cavity, normalised peak curvature of the boundary, and the desired accuracy (in digits) in almost equal manner. We emphasise this because, as a rule, the published works where BIEs are used ignore the last two parameters and blindly rely on the "rule-of-a-thumb" of taking 10 mesh points per wavelength.

It should be noted that the Muller BIE method relates to the so-called analytical regularisation techniques, and explicitly inverted part of the full problem can be identified as zero-contrast one—see Nosich (1999) for a review of such techniques.

This powerful method has been already applied to the accurate 2-D analysis of modes in elliptic (Boriskina et al. 2004a), periodically corrugated circular (Boriskina et al. 2004b), rounded triangular and rectangular (Boriskina et al. 2005), and notched-circular (Boriskina et al. 2006) passive dielectric cavities, including very high-Q-factor WG modes (see Figs. 4, 5). PMs made of three and four identical microdisks with optically coupled WG modes were studied in Boriskina (2006). It has been found that the Q-factors of the WG supermodes strongly depend on the rim-to-rim spacing and can be much higher than for a

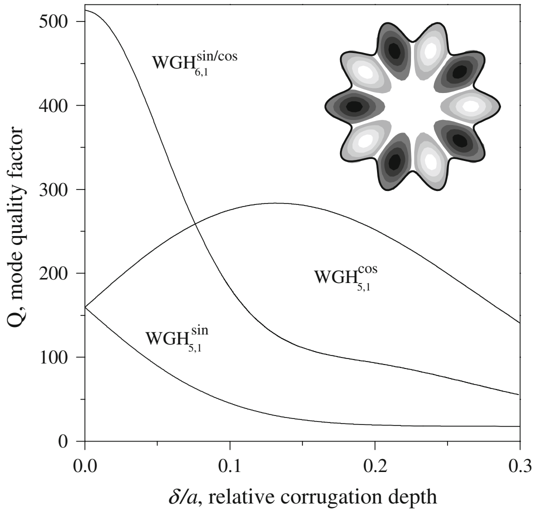


Fig. 4 Q-factors of the deformation-matched and mismatched $(H_z)_{m,1}$ modes in a "flower-shape" GaAs cavity as a function of sinusoidal corrugation depth normalised to the radius. $\alpha_{\mathrm{eff}} = 2.64$, $\lambda = 1.55\,\mu\mathrm{m}$

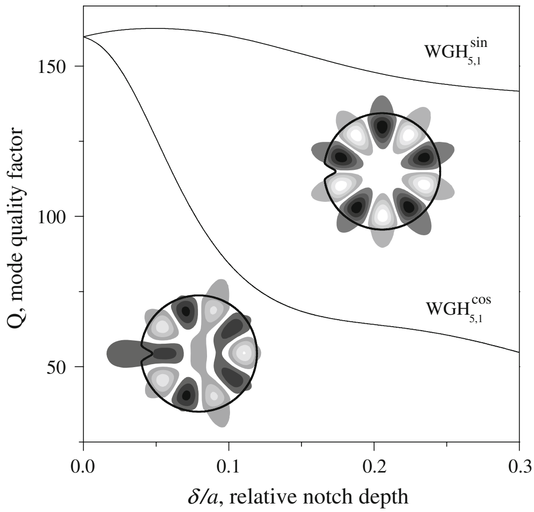


Fig. 5 Q-factors of the deformation-matched and mismatched $(H_z)_{5,1}$ modes in a GaAs notched circular cavity as a function of the notch depth normalised to the cavity radius; $\alpha_{\mathrm{eff}} = 2.64$, $\lambda = 1.55\,\mu\mathrm{m}$

stand-alone disk. Recently this study has been extended to the microdisk coupled-resonator optical waveguide configurations in Boriskina (2007).

Vector-field modal analysis of fully 3-D cavities with Muller BIEs remains a topic for future studies. It will clarify, for instance, the effect of the disk rim profile on the WG-mode Q-factors that is intractable within a 2-D simulation.

As it has been emphasised in Nosich et al. (2006), from the viewpoint of the mathematical theory of linear electromagnetic boundary-value problems (BVP) and the equivalent VIEs and BIEs, classification of non-circular dielectric cavities as stable, unstable and chaotic is senseless. Any passive open resonator possesses a discrete albeit infinite set of complex-valued natural frequencies, k_s, which are the generalised eigenvalues of BVP. Each of them can be only of finite multiplicity and depends in piece-continuous manner on the geometry and refractive index (continuity may be lost only if two eigenvalues coalesce). Each simple eigenvalue generates a unique eigenfunction—this is the modal field characterised by two corresponding vector-functions, $\vec{E}_s$ and $\vec{H}_s$. In this sense, all WG and "bow-tie" or "scarred" modes are equally stable however differ in the values of their frequencies and Q-factors. In the linear-problem case, any "chaos" appears only when we try to simplify the task and superimpose a rough (and scalar) ray-tracing approximation on the original wave-like (and vector) formulation in terms of Maxwell's equations.

Mathematically, there is a well-established understanding that both Maxwell and Helmholtz equations have what is called an elliptic behaviour. This means that the whole modal field must be found simultaneously using the boundary conditions on the entire surface of the cavity. In contrast, the rays emerge as solutions to the eikonal equation, which is an extreme form of the parabolic equation that, in its turn, approximates exact ones only for the field behaving as a locally plane wave. Parabolic equation, indeed, allows a huge computational simplification as it can be solved step-by-step downstream from a start point. For the rays this simplification goes even further and reduces to the Snell law of the specular reflection from the boundary of cavity. This is where the chaos emerges in the form of chaotic trajectories of rays. Therefore the chaos is the cost of pleasure—in the same manner as the branching of the habitat of complex eigenfrequencies is the cost of switching from the accurate 3-D to an approximate 2-D BVP for arbitrary open cavity.

10 Modelling: from passive cavity to cavity with active region

Until recently, linear modelling of microdisk and other microcavity lasers has implied the calculation of the natural modes of the cold cavities, i.e. *passive* open dielectric resonators. Mathematically this means solving the eigenvalue problem for the complex-valued natural frequencies. On finding these frequencies, k_s, the modes with the largest Q-factors (i.e., the smallest values of Im k_s) are associated with lasing.

It is easy to see, however, is that the lasing phenomenon is not addressed directly through the Q-factor—the specific value of threshold gain that is needed to force a mode to become lasing is not included in the formulation. An immediate practical consequence of this is a well-known inability of the Q-factor theory to explain why in the stadium-shape cavity the lasing occurs on the "bow-tie" modes, whose Q-factors are much lower than those of the WG-like modes. Trying to answer this question, researchers had to resort to complicated non-linear descriptions (Tureci and Stone 2005).

Remarkably, this gap in the characterisation of lasers can be filled in by a relatively simple modification of the formulation of the linear electromagnetic problem. Namely, introduction of macroscopic gain, say γ, into the cavity material enables one to extract not only the frequencies but also the threshold material gains as eigenvalues. The most convenient way for doing this is through the *active* imaginary part of the complex refractive index ν: if the time dependence is assumed as $e^{-i\omega t}$, then $\nu = \alpha - i\gamma$, $\alpha, \gamma > 0$. Such a *lasing eigenvalue problem* (LEP) was suggested in Smotrova and Nosich (2004), which was a development of an earlier conference paper. Note that general properties of the LEP eigenvalues have been established for arbitrary-shape open resonators (Smotrova et al. 2005a) and show that (i) all $\gamma_s > 0$ (no thresholdless lasing is possible); (ii) eigenvalues form a discrete set on the plane (k, γ); (iii) each eigenvalue continuously depends on the resonator geometry and refractive index, and may disappear only at infinity on the plane (k, γ). The gain per unit length, the traditional quantity in the descriptions of the Fabry-Perot cavities, is $g = k\gamma$, where $k = \omega/c = 2\pi/\lambda$. To link the LEP with the more traditional eigenfrequency (i.e., Q-factor) problem, one can keep in mind that each discrete eigenfrequency is a function of the gain parameter, γ. Hence, one may look for a specific value, γ_s, that provides

Im $k_s(\gamma_s) = 0$, and consider this as the threshold of lasing at which the radiation losses are balanced exactly with the macroscopic gain of active medium. Note also that looking for a mode Q-factor in an active cavity (like in Ripoll et al. (2004)) makes little sense, because it may become arbitrarily large depending on the nearness of the gain γ to the threshold value, γ, at which the Q-factor is infinite—see also Kerker (1979). Still Ripoll et al. (2004) is an important paper as it establishes the links between γ and microscopic medium parameters based on the two-level model, and also the pumping.

In fact, a suggestion to introduce the material gain in the optical-field problem and use it to characterize the threshold can be met in several papers published in the 1970–1990s. For instance, it was expressed in Vlasov and Skliarov (1977) for the modes of a dielectric rod with highly-reflecting facets. More precisely, it was suggested to look for the value of Im ε ($\varepsilon = \nu^2$), which brought the imaginary part of complex eigenfrequency to zero. The same approach was used in Noble et al. (1998) when studying the modes of the VCSEL-type configurations. Only within the last few years, has a more systematic search for a threshold value of gain has been proposed—see Campenhout et al. (2005) for the photonic-crystal membrane laser and Sun et al. (2007) for an ABR-assisted disk laser. Unfortunately, in these papers the approximate optical modelling techniques did not allow clear estimation of the nearness of the solutions obtained to accurate ones.

In contrast, the study of Smotrova and Nosich (2004) dealt with accurate LEP-based quantification of the thresholds of both WG and non-WG modes in the active circular resonator as a 2-D model of a pumped microdisk. Essential quasi-3-D features were further provided by accounting for the multiple-value character of the thin-disk effective index and for its dispersion (Smotrova et al. 2005a). The validity of the LEP approach was supported by good agreement with published experimental data for the frequencies of lasing in a thick disk including higher-order modes.

A non-uniform distribution of gain across the disk, due to either shaped pump beam or shaped electrodes cannot be accounted for in the passive cavity model but is easily accounted for in LEP. To this end, one has to introduce the gain γ only inside the active region and impose an additional set of the field tangential components continuity conditions on the boundary of this region. In Smotrova et al. (2005a), such a 2-D LEP analysis was done for microdisks with active regions shaped as (i) a circle centred inside the disk and (ii) a ring adjacent to the disk rim.

For example, if the active region radius shrinks to $0.6a$ for a GaAs/InP disk, the threshold of the $H_{(z)20,1}$ mode jumps up by 10^5 times. In contrast, a ring-shape active region may be as narrow as $0.2a$ and still provide the same value of the material gain threshold of the same mode as in the uniformly active disk (Fig. 6). Therefore, the reduction of the pump power reported in Rex et al. (2001) can be attributed to the proportional reduction of the illuminated area. Thus the intuitive idea of the importance of "spatial matching" between the active region and the modal field pattern is incorporated into the LEP automatically.

In Smotrova et al. (2006a), a twin-disk PM laser has been studied, based on the reduction of the LEP to a Fredholm second kind matrix problem. The analysis has shown that, due to the coupling, each pair of degenerate WG modes splits into four orthogonal coupled modes (i.e., supermodes) of different symmetry classes. The most interesting result is that, for each of the supermodes, careful tuning of the distance between the disks may provide a threshold, which is lower than for a single cavity. The directivity of light emission from PMs is generally greater than 2 as the patterns display a small number of identical intensive beams—e.g., from eight to two for a twin-disk PM laser. Further, similar studies were performed for a two-disk PM with one active (e.g., selectively pumped) and another passive cavity (Smotrova et al.

Fig. 6 Thresholds of the $(H_z)_{m1,0}$ modes in a GaAs circular cavity with the ring-like active region as a function of its inner radius normalized to the cavity radius; $\alpha_{eff} = 2.64$, $\lambda = 1.55\,\mu m$

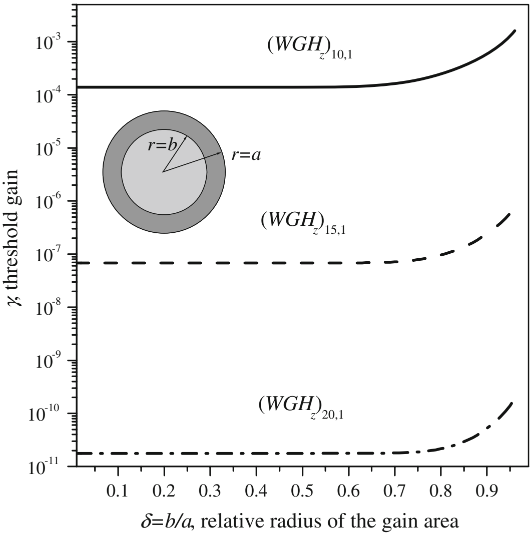
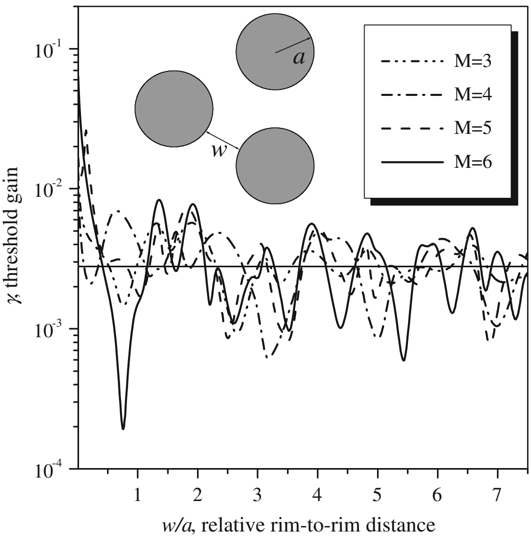


Fig. 7 Threshold gains versus the rim-to-rim airgap width normalized to the elementary cavity radius, for the lasing supermodes of the $(H_z)_{6,1}$ type of the maximally symmetric field class. M is the number of microdisks in PM. The straight line is the threshold of the corresponding mode in the single circular cavity; $\alpha_{eff} = 2.64$, $\lambda = 1.55$



2005b) and for PMs arranged as cyclic arrays of M identical active microdisks (Smotrova et al. 2006b).

For the latter PMs, a remarkable symmetry-assisted reduction of thresholds has been found for the supermodes built on the WG modes in individual cavities (see Fig. 7), if the distance between the adjacent disks is properly tuned. This phenomenon is similar to the increase of Q-factors in a passive cavity reported in Boriskina (2006). Thus, the idea of Evans and Holonyak (1996) was generally right although they had apparently selected a wrong distance ($w/a = 0.36$) between the WG-mode resonators in a cyclic PM laser. The resonance reduction of threshold is generally located nearer to $w/a \approx 1$ although this value may be smaller for the higher-index WG modes.

When using the WG modes, precise tuning to the reduced-threshold configuration can be difficult engineering task. Therefore the non-resonant threshold reduction in the same cyclic PM reported in Smotrova et al. (2006c) for the case of an even number of disks looks more attractive for realization. It takes place for small sub-wavelength disks that are not able to support WG modes at all. However, if the monopole modes in adjacent cavities, $(H_z)_{0,1}$, have opposite phases, then the corresponding π-type supermode has reduced threshold due to efficient cancellation of the radiation losses. The same effect takes place, both for even and odd M, for the π-type supermode built on the $(H_z)_{1,1}$ modes in individual cavities. Realization of such a PM laser seems to be easier due to the simple engineering rule: the larger the disk number and the smaller the rim-to-rim distance, the lower the threshold.

Linear modelling of lasing thresholds of a microdisk equipped with single or several QDs (a device reported by Zwiller et al. (2003b)) is another example of a problem not tractable with a passive cavity model, however accessible through the LEP analysis. Here, one can expect that a QD must not be too small and must be located not too far from the rim of the disk to provide a reasonably low linear threshold.

Of course, the LEP formulation can be applied to the analysis of modal thresholds and frequencies in the models of any dimensionality for any kind of laser and combined with any specific analytical or numerical technique for the field description. For instance, in Byelobrov and Nosich (2007) it is used in the classical 1-D model of a VCSEL with a QW and two distributed Bragg reflectors (DBRs) and accurately quantifies the lowering of thresholds of those modes whose frequencies match the DBR rejection bands. Another example is Chu and Otsuka (2007), where the authors use the formulation that is essentially similar to LEP to study a two-mirror laser resonator with one concave and another flat mirror with the aid of parabolic-equation technique; the active region is a selectively pumped crystalline slab attached to the flat mirror. Numerical results show importance of good overlap of the pumped area with the desired mode pattern. The third example is a very recent paper (Manolatou and Rana 2008) where a family of surface-plasmon-assisted nanolasers is considered using FDTD solver and LEP-like formulation: threshold currents are found from the condition of zero modal losses.

Finally, it is worthy of note that the finite spectral width of the photoluminescence of the active region can be taken into account in the LEP in the same manner as the dispersion of the effective refractive index—see Smotrova et al. (2005a). Namely, one should introduce the function $\bar{\gamma}(\lambda) = \gamma f(\lambda)$, where $0 < f(\lambda) \leq 1$ is a known function, e.g. a Gaussian, and coefficient γ is the eigenvalue. It is clear that, in such a formulation, only those lasing modes whose frequencies spectrally match the photoluminescence band, i.e. the band where $f(\lambda)$ is reasonably close to 1, will keep low thresholds.

11 Conclusions

We have reviewed recent trends in the design of microcavity lasers and looked at them from the viewpoint of accurate modelling in a linear approximation. To obtain unambiguous interpretation of calculated data and effects it is mandatory to have a highly reliable modelling tool. Such a tool must somehow tackle arbitrarily curved boundaries, use rigorous boundary conditions, and accurately account for the open host space; hence ray tracing and time domain numerical codes are not good candidates, and integral equation ones are more promising. They must be still free of mathematical defects that may come from the nature of integral equations or the scheme of their discretisation. Otherwise extremely high-Q modes, which start lasing first due to ultra-low thresholds in the active cavity, cannot be accessed in stable and accurate manner. All these criteria are satisfied when using the technique based on the Muller BIEs.

Finally, we emphasise that to analyse the lasing in direct manner, though still within a linear formulation, one has to switch from a passive cavity to the cavity with material gain in the active region, i.e., to study a LEP. This enables one to account for the precise shape of the active region and, additionally, for the frequency dependence of the refractive indices and material gain. However in this case, analysis of Q-factors should be replaced with analysis of the threshold values of material gain. Therefore, the LEP can be called a "warm-cavity" model of laser as it takes into account, although in an averaged manner via the macroscopic concept of an "active" imaginary part of refractive index, the presence of light-emitting carriers in semiconductor material.

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