Threshold reduction in a cyclic photonic molecule laser composed of identical microdisks with whispering-gallery modes

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Lasing modes in cyclic photonic molecules (CPMs) composed of several identical thin semiconductor microdisks in free space are studied in a linear approximation. Maxwell's equations with exact boundary conditions and the radiation condition at infinity are considered as a specific eigenvalue problem that enables one to find natural frequencies and threshold gains. It is demonstrated that careful tuning of the distance between the disks in CPMs is able to drastically reduce the lasing thresholds of the whispering-gallery modes having small azimuth indices. © 2006 Optical Society of America

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Semiconductor microdisks equipped with quantum wells and quantum dots are being intensively investigated now due to their fascinating lasing properties and their possible applications in optoelectronic circuits that enable large-scale integration. The main features of these lasers are periodically spaced frequencies of lasing, very low thresholds, and predominantly in-plane light emission. For applications, high-performance cavities with ultralow threshold and directive emission are desirable. Recently, cavity structures more complex than those with a single microcavity have been proposed to enhance the light output as a result of optical-field coupling. Photonic molecules (PMs) composed of circular, linear, and square arrays of microrings, microdisks, and microspheres have been reported, and optical whispering-gallery (WG) mode splitting into multiplets of "supermodes" with closely spaced frequencies has been detected.[1–5]

Our goal is the accurate study of the lasing modes of the cyclic PMs (CPMs) of optically coupled microdisks in a linear approximation. To this end, we will use the lasing eigenvalue problem (LEP), specifically tailored to extract not only the lasing frequencies but also the corresponding threshold gains from the linear field equations.[6] We have already applied the LEP to one-disk lasers with radially nonuniform gain[7] and to two-disk PM lasers with uniformly and selectively activated disks.[8,9]

Suppose that a CPM is composed of M identical microdisk cavities located in the same plane in free space with their centers in the vortices of a regular polygon (Fig. 1). Each disk has thickness d, radius a, and real-valued refractive index α, and the distance between the adjacent disks is w. Time dependence is implied as $e^{-i\omega t}$, and the free-space wavenumber is $k=\omega/c=2\pi/\lambda$, where λ is the wavelength. If d is only a fraction of λ, the 3D optical field problem can be reduced with the effective index method[8] to a 2D one in the disks' plane.

Then, two different polarizations can be treated separately with the aid of a function U, which is either the E_z or H_z field component. The LEP statement implies that U must satisfy the 2D Helmholtz equation, where inside the disks the bulk refractive index α is replaced with the complex-valued parameter $\nu=\alpha_{\text{eff}}-i\gamma$, otherwise $\alpha=1$. Here $\gamma>0$ is the material gain, which is assumed to be uniform across the disks. At the disk rims the continuity conditions for the tangential field components hold, and the Sommerfeld radiation condition is imposed at infinity. We seek the eigenvalues as pairs of real-valued dimensionless parameters, (κ,γ), where $\kappa=ka$ is the normalized frequency. Note that the gain per unit length (the traditional quantity in the Fabry–Perot

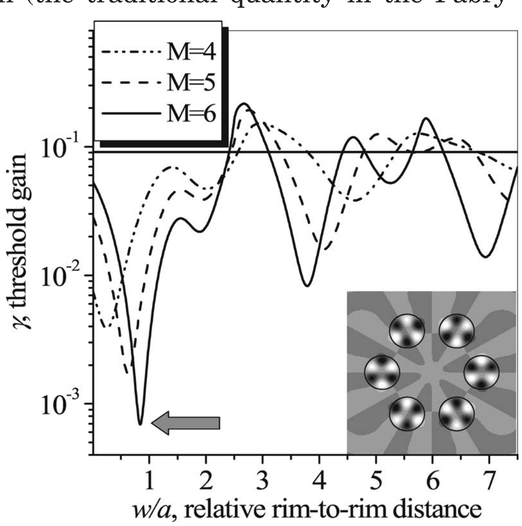

Fig. 1. Threshold gains versus the relative rim-to-rim distance for the H_z-polarized lasing supermodes in PMs of the $(H_z)_{3,1}$ type of the maximally antisymmetric field class. M is a number of microdisks in the PM. The straight line is the threshold of the corresponding mode in a single cavity. The inset shows the near-field pattern at the distance providing the minimum threshold of the supermode in the PMs of six disk cavities.

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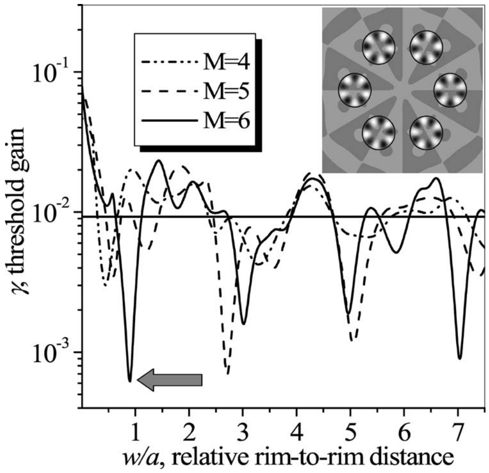

Fig. 2. Same as in Fig. 1 for the lasing supermodes of the $(H_z)_{5,1}$ type.

laser cavities measured in inverse centimeters) can be obtained as $g = k\gamma$.

A CPM of M microdisks has M-fold symmetry. Therefore all natural modes split into $2M$ classes according to parity with respect to the symmetry lines. The circular boundaries of the cavities enable efficient use of the method of separation of variables. In line with the method, we expand the field function inside each cavity in terms of angular functions. In free space the field is assumed to be a superposition of similar expansions generated by all resonators and satisfying the radiation condition at infinity. After applying the boundary conditions, using the addition theorem for cylindrical functions, and satisfying the requirements of symmetry or antisymmetry, we obtain the infinite matrix equations for all symmetry classes. Any of them can be written as $[I + G(\kappa, \gamma)]X = 0$, where $X = \{x_m\}_{m=(0)1}^{\infty}$ is the vector of expansion coefficients, I is the identity operator, and $G = \{G_{mp}\}_{m,p=(0)1}^{\infty}$ is the so-called compact operator. Therefore the equations obtained are the second kind Fredholm ones. Then the search for the LEP eigenvalues is reduced to finding zeros of the determinants of truncated equations, $\mathrm{Det}^{(N)}[I + G(\kappa, \gamma)] = 0$, and convergence to the exact eigenvalues of infinite matrices is guaranteed when the truncation number N is increased. We have computed the eigenpairs (κ, γ), which are lasing frequencies and thresholds, with a two-parameter secant-type iterative method.[7] In computations we kept the resulting accuracy of computations at the level of 10^{-6} and assumed a GaAs–InP system, $\lambda = 1.55$ nm, $\alpha = 3.374$, and $d = 0.2\ \mu$m, so that $\alpha_{\mathrm{eff}} = 2.63$ (this value may be somewhat overestimated for the modes with small azimuth indices).

In Figs. 1–4 we present the dependences of the lasing thresholds on the relative rim-to-rim distance, w/a, for the supermodes $(H_z)_{m,1}$ with $m = 3, 5, 7, 10$ of the maximally antisymmetric field class in CPMs of 4, 5, and 6 microdisks. Here the first index corresponds to the number of field variations along the elementary disk rim, and the second is the same along the radius. As seen, if the separation is small ($w < 0.2a$), then the thresholds of the maximally antisymmetric WG supermodes grow when they are brought together. However, if the separation is comparable to the disk radius, the thresholds can drop to significantly lower values than for those of an isolated disk. Such a reduction of threshold is observed in narrow bands, depends on m, and needs fine tuning of separation w. Increasing the number of microdisks in a CPM leads to a further lowering of the minimum values of thresholds, although in a narrower band of w values. A very large separation ($w > 3$) reduces the radiative coupling, and thresholds oscillate close to their values for the stand-alone cavity.

For the WG supermodes of smaller azimuth indices, such as $m = 3, 5$, smaller threshold reductions relative to the one-disk value are observed in several bands of the w/a variation. This is explained by weak field confinement in each disk. If the index is too large, say $m = 10$, then the field confinement is very strong, and threshold reductions are again small and found in several bands of the w/a variation. Only for the supermodes of intermediate indices, say $m = 7$ in our case, one can see a single band of the most significant threshold reduction.

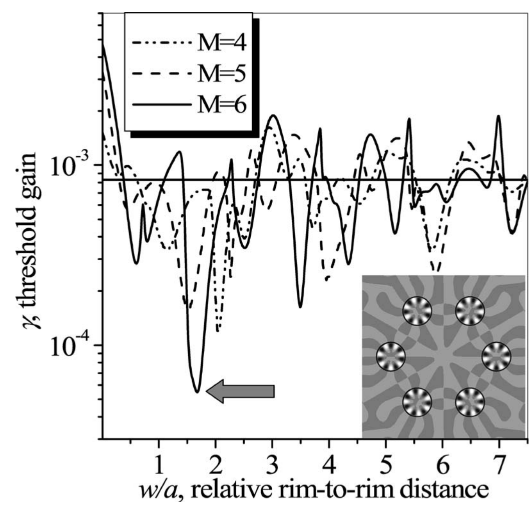

Fig. 3. Same as in Figs. 1 and 2 for the lasing supermodes of the $(H_z)_{7,1}$ type.

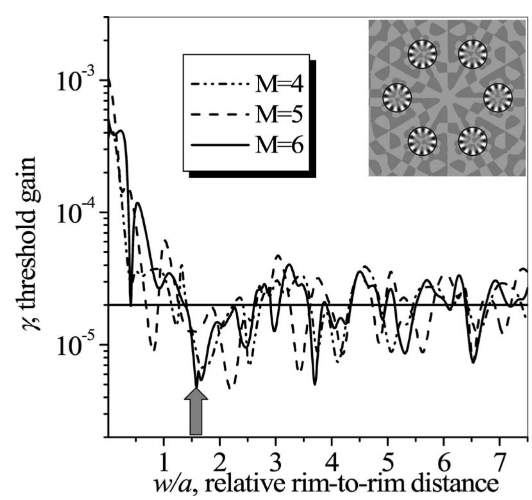

Fig. 4. Same as in Figs. 1–3 for the lasing supermodes of the $(H_z)_{10,1}$ type.

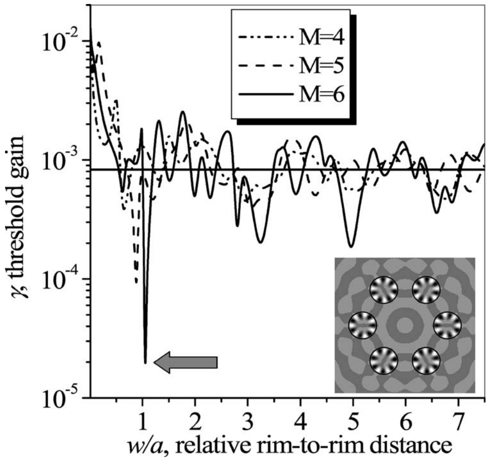

Fig. 5. Same as in Fig. 3 for the lasing supermodes of the $(H_z)_{7,1}$ type of the maximally symmetric field class.

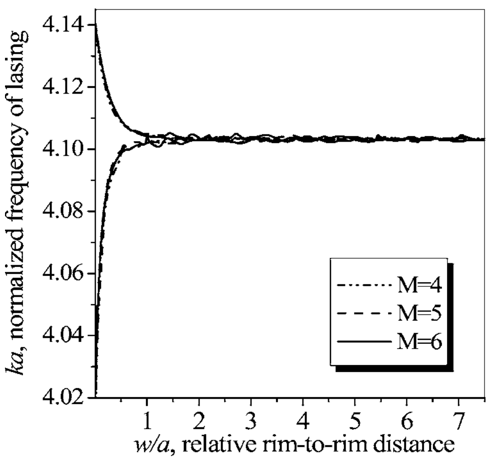

Fig. 6. Lasing frequencies versus the relative rim-to-rim distance for the supermodes of the $(H_z)_{7,1}$ type of the maximally antisymmetric (blueshifted) and maximally symmetric (redshifted) field classes.

A similar behavior of lasing thresholds is observed for the lasing supermodes of the maximally symmetric field class (see Fig. 5).

In Fig. 6 we present the w/a dependences of the lasing frequencies of the maximally antisymmetric and maximally symmetric WG supermodes $(H_z)_{7,1}$. They show that these supermodes are closely spaced if w/a is large and that they obtain a redshift or blueshift, respectively, if this value becomes smaller than 1.

The near fields of the coupled modes in a CPM can vary greatly depending on the mode type and the separation between the disks. In the insets in Figs. 1–5 we present the field portraits corresponding to the minima of thresholds for a six-disk CPM (marked by arrows). They demonstrate that low-threshold supermodes have the fields locked inside elementary disks and that little leakage occurs to free space.

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