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PRINCIPAL AND HIGHER ORDER MODES OF MICROSTRIP AND SLOT LINES ON A CYLINDRICAL SUBSTRATE

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1. INTRODUCTION

Microstrip and slot antennas, and transmission lines which conform to cylindrical non-planar substrates, may be of significant interest in many applications. In previous papers [1], [2], the waveguide properties of a microstrip line on a circular cylindrical substrate were characterized under the approximations of a quasistatic model. A dynamic analysis has been presented in [3]. A similar analysis has been carried out in [4] for a pair of coupled microstrip lines. The results presented up to now refer only to the dominant principal waves of these guides, while results on other principal and higher order modes remain unpublished. However, as the frequency of operation is increased, microwave and millimeter wave integrated circuits on curved substrates call for the development of a detailed description of the modes supported by such structures.

In this paper we analyze the same structure as in [3], the geometry of the cross section being shown in Fig. 1. We do not introduce any restrictions on wavelength or on angular width of the strip, thus obtaining the possibility to consider an analogous slot line on cylindrical substrate with the same treatment. The method of analysis follows [5] and [6], which was developed for a partially screened circular dielectric core. It is based on reducing the initial problem first to dual series equations, and then to a regularized Fredholm-type system of linear algebraic equations of the second kind. The key idea of regularization is to make use of the so-called Riemann-Hilbert technique, the mathematical details of which may be found in [7]. This approach enables one to obtain proof of the existence of a discrete eigenvalue spectrum in the same way as for a partially screened core [5], [6]. Even more importantly, it results in a highly effective numerical algorithm with a guaranteed accuracy, in contrast to the solutions obtained by most of the moment, or Galerkin, methods. This provides a way to check the validity of the approximate results of [1] and [2] for the dominant mode characterization, as well as to investigate dispersive behavior of other principal modes (without cutoff), and higher order modes of these lines.

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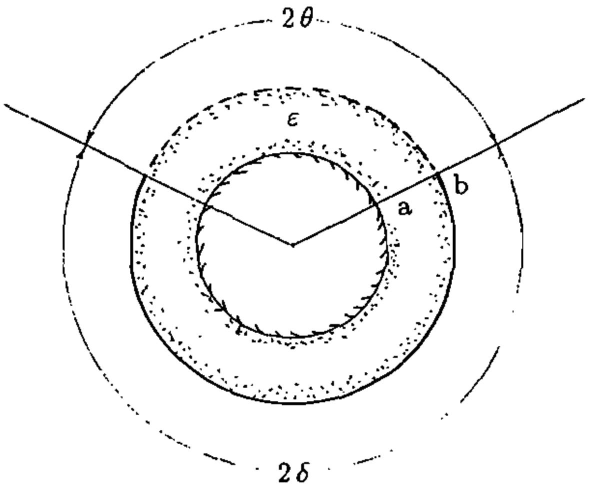

FIGURE 1: Geometry of the problem

2. SPECTRAL PROBLEM: FORMULATION AND ANALYSIS

Seeking the solution of Maxwell equations with corresponding boundary conditions in the form of a normal guided wave that depends on z and t as $e^{(ihz-i\omega t)}$, one comes to the cross-sectional eigenvalue problems for a 2-D Helmholtz equation

$$(\Delta + \gamma_j^2)\overline{W}^{(j)}(\bar{r}) = 0 \tag{1}$$

where the components of $\overline{W} = (U, V)$ represent longitudinal field components E_z and H_z, and $\gamma_j^2 = k^2\varepsilon_j - h^2$ with $j = 1, 2$, denoting the substrate ($\varepsilon_1 = \varepsilon$) or free space ($\varepsilon_2 = 1$) regions, respectively. The boundary conditions at $r = a$ yield

$$L_E \overline{W}^{(1)}\Big|_{r=a} = 0 \ , \tag{2}$$

while at $r = b$

$$\left[L_E \overline{W}^{(j)} \right]_{j=1}^{2} \Big|_{\partial D} = 0 \tag{3}$$

$$\left[L_H \overline{W}^{(j)} \right]_{j=1}^{2} \Big|_{\partial D} = 0 \tag{4}$$

$$L_E \overline{W}^{(1,2)}\Big|_{\partial M} = 0 \ , \tag{5}$$

where

$$L_E = \begin{bmatrix} 1 & 0 \\ \dfrac{h}{\gamma_j^2 r}\dfrac{\partial}{\partial\phi} & \dfrac{-k}{\gamma_j^2}\dfrac{\partial}{\partial r} \end{bmatrix} \qquad L_H = \begin{bmatrix} \dfrac{-k\varepsilon}{\gamma_j^2}\dfrac{\partial}{\partial r} & \dfrac{h}{\gamma_j^2 r}\dfrac{\partial}{\partial\phi} \\ 0 & 1 \end{bmatrix} ,$$

and circular arcs $\partial D = (r = b, \ |\phi| < 0)$ and $\partial M = (r = b, \ \theta < |\phi| \leq \pi)$ denote cross-sectional contours of the slot and the strip, respectively.

Because of the sharp edges of the strip, we have to introduce the condition of local energy limitation

$$B \int \left(|k\overline{W}|^2 + |\nabla \overline{W}|^2 \right) d\bar{r} < \infty \tag{6}$$

for any compact domain $B \subset R^2$, e.g., the one containing sharp edges. This inequality is equivalent to the well known Meixner edge condition which specifies the field's behavior as functions of coordinates in the vicinity of the edge.

To have a complete formulation of the problem, one has to impose some condition at infinity in the cross section, due to the open nature of the guide. In the conventional theory of guided waves, the condition of exponential decay is usually applied. However, except for the surface mode, other modes are eliminated, although leaky modes and other complex modes may also be of interest. The most general condition of this kind is derived from the fact that Green's function of Eq. (1) is a Hankel function of the first kind. Taking into account the addition theorem for cylindrical functions, one comes to the requirement that, outside a finite-radius circle (e.g., $r = b$), the field has to be expandable in the series

$$\overline{W}(\bar{r}) = \sum_{n=-\infty}^{\infty} (a_n, b_n) H_n^{(1)}(\gamma r) e^{in\phi} , \tag{7}$$

where $\gamma \equiv \gamma_2 : \gamma_2^2 = k^2 - h^2$.

The basic problem in waveguide simulation is to find the complex eigenvalues for h, which generate nontrivial eigenfunctions $\overline{W}(r)$ satisfying equations (1)-(7). When obtaining the value of h, one can compute the mode field components, impedance, and even losses within the modified perturbation theory [8].

To discretize this problem, we use the expansion of fields in a Fourier series over angular functions, and take into account Eqs. (1), (2), and (7).

$$\begin{aligned}
\left(U^{(1)}, V^{(1)}\right) &= \sum_n (A_n, B_n) \left[J_n(\gamma_1 z) + \left\{ \frac{J_n(\gamma_1 a)}{H_n(\gamma_1 a)}, \frac{J_n'(\gamma_1 a)}{H_n'(\gamma_1 a)} \right\} H_n(\gamma_1 z) \right] e^{in\phi} \\
\left(U^{(2)}, V^{(2)}\right) &= \sum_n (A_n', B_n') H_n^1(\gamma_2 z) e^{in\phi}
\end{aligned} \tag{8}$$

Substituting this series into the remaining boundary condition (Eqs. (3)-(5)), we come to the dual series equations for the expansion coefficients, which are valid on ∂D and ∂M of the circle $r = b$. After some manipulation based on the assumption that the series allows term-by-term differentiation, we obtain the coupled pair of dual series equations of canonical form:

$$\begin{cases}
\sum_n p_n |n| e^{in\phi} = \sum_n \left(\Delta_n p_n + \alpha n \varepsilon_n^E \mu_n \right) e^{in\phi}, & |\phi| < \theta \\
\sum_n p_n e^{in\phi} = 0, & \theta < |\phi| \le \pi
\end{cases}$$

$$\begin{cases}
\sum_n \mu_n |n| e^{in\phi} = \sum_n \left(\beta n \varepsilon_n^H p_n + \Delta_n^H \mu \right) e^{in\phi}, & \theta < |\phi| \le \pi \\
\sum_n \mu_n e^{in\phi} = 0, & |\phi| < \theta
\end{cases} \tag{9}$$

where we denote

$$\alpha = \frac{1}{2} ihkb^2 (\varepsilon - 1) x^{-1} (\varepsilon + 1)^{-1} ; \qquad \beta = ihkb^2 (\varepsilon - 1) x^{-1} y^{-2} ;$$

$$\varepsilon_n^E = 1 + (x^2+y^2)F_n(f_n - F_n)^{-1}y^{-2}\,; \qquad \varepsilon_n^H = 1 + (x^2+y^2)f_n(F_n - f_n)^{-1}x^{-2}\,;$$

$$\Delta_n^E = |n| - \frac{(x^2+y^2)}{2(\varepsilon+1)}\left[\varepsilon\phi_n - F_n - \frac{n^2h^2k^2b^4(\varepsilon_1)^2}{x^4y^4(f_n-F_n)}\right]\,; \tag{10}$$

$$\Delta_n^H = |n| + \frac{(x^2+y^2)f_nF_n}{f_n - F_n}\,; \quad x = \gamma_2 b\,; \quad y = \gamma_1 b\,; \quad y_2 = \gamma_1 a\,;$$

$$\phi_n = \beta_n'(y)\,[y\beta_n(y)]^{-1}\,; \qquad F_n = H_n'(x)\,[x H_n(x)]^{-1}\,;$$

$$\beta_n(y) = -\frac{H_n(y)}{H_n(y_2)}J_n(y_2) + J_n(y)\,; \qquad \beta_n'(y) = -\frac{H_n'}{H_n(y_2)}J_n(y_2) + J_n'(y)\,.$$

Equations of this type are known as ones with a trigonometric kernel, due to the well known expansion of angular functions in terms of trigonometric functions.

After application to any limited domain (e.g. the substrate region), the last of the initial conditions, Eq. (6), yields:

$$\sum_{(n)} |\mu_n|^2\,|n+1| < \infty\,; \qquad \sum_n |p_n|^2\,|n+1| < \infty\,. \tag{11}$$

The left-hand sides of each of the dual series equations (9) may be shown to be equivalent to the boundary value problem called the Riemann problem (sometimes referred to as the Hilbert problem). The solution to this Riemann-Hilbert problem may be found in [7]. When applied to our case, this solution permits us to carry out the partial inversion procedure because of the fact that unknown coefficients are also present in the right-hand side. In terms of the discretized problem under consideration, this results in an infinite system of linear algebraic equations (see [7] for details):

$$\begin{aligned} p_n &= \sum_n \left(A_{mn}^{11}p_n + A_{mn}^{12}u_n\right) \\ \mu_m &= \sum_n \left(A_{mn}^{21}p_n + A_{mn}^{22}\mu_n\right), \quad m = 0, \pm1, \dots, \end{aligned} \tag{12}$$

where

$$\begin{aligned} A_{mn}^{11} &= \Delta_n^E\tau_{mn}(u)\,; \qquad A_{mn}^{12} = \alpha n\varepsilon_n^E\tau_{mn}(u)\,; \\ A_{mn}^{21} &= \beta n\varepsilon_n^H\tau_{mn}(-u)(-1)^{m+n}\,; \qquad A_{mn}^{22} = \Delta_n^H\tau_{mn}(-u)(-1)^{m+n}\,; \\ \tau_{mn}(u) &= 2^{-1}(m-n)^{-1}\left[P_{n-1}(u)P_m(u) - P_n(u)P_{m-1}(u)\right]\,; \qquad u = \cos\theta\,; \end{aligned} \tag{13}$$

and $P_n(\pm u)$ are Legendre polynomials.

Due to the well known asymptotic behavior of Legendre polynomials, the solutions of (12) may be shown to behave as $0(|m|^{-3/2})$ for large $|m|$. Thus, the conditions (11) are also satisfied. But the most important thing is that Eqs. (12) are of the Fredholm-type system of the second type, as the series

$$\sum_{m,n} |A_{mn}|$$

is convergent. This fact guarantees numerical convergence of computations when approximating the roots of the equation

$$\det(1 - A(h)) = 0$$

by means of the roots of its truncated analog. To obtain any desired accuracy, one has to increase the order of truncation N_t appropriately. The following simple empirical rule has been verified: 0.1% accuracy may be achieved by taking

$$N_t = \text{entire}(\max(x, y)) + 5$$

for each of the four blocks of the whole $A(h)$ matrix.

3. CLASSIFICATION OF MODES

As can be seen in Fig. 1, the structure under investigation resembles two different transmission lines, depending on the strip/slot widths relation. One of them is a Goubau line as $\theta/\delta < 1$, and the other is a coaxial waveguide with dielectric filling as $\delta/\theta < 1$. A narrow strip or a narrow slot plays the role of perturbation factor for each of these guides, shifting the mode spectrum points $h_j(\theta)$ in a continuous manner as functions of θ (or δ). It is well known that there are as many as three modes in a Goubau line, having no cutoff at low frequencies, i.e. the axially symmetric E_{00} mode and two orthogonally polarized HE_{11} modes. All these modes are called principal modes (the term was introduced by Sommerfeld), in contrast to higher order modes having finite cutoff frequencies. The E_{00} mode is also called dominant, for the much slower velocity than that of HE_{11} modes. Thus, the modal spectrum of the Goubau line contains at least two real points, $h_{E_{00}}$ and $h_{HE_{11}}$, and an infinite number of complex points. All the points corresponding to axially nonsymmetric modes are of double multiplicity (including $h_{HE_{11}}$), due to the symmetry of the cross section.

The modes of the coaxial guide are also well known. The modal spectrum contains a point $h_{T_0}i = k(\varepsilon)^{1/2}$, at most finite number of real points corresponding to guided waves and infinite number of imaginary points generating purely decaying modes. The same considerations regarding degeneracy are valid, due to the symmetry of the cross section.

We must also take into account the modes of external domain of a circular perfectly conducting cylinder. These are produced by zeros of Hankel functions and their derivatives of argument γb, known to be the complex values. In addition, there is one more point of the spectrum $h = k$ (coinciding with the branch point), which relates to the axially symmetric T_0^e mode of the TEM type.

It must also be noted that in these guides there exist infinite numbers of azimuthal mode families. By introducing a strip or a slot, we obtain only two families of modes, in the sense of symmetry relating to the reference plane. It is convenient to denote them as E_z^+/H_z^- and E_z^-/H_z^+, due to the even or odd character of longitudinal fields. A narrow strip or plot perturbs degenerated points of the spectrum in a different manner for each of these families, thus splitting them into two separate ones. But what is more important, a strip or a slot produces a new mode as a point of the spectrum. For a narrow strip $(\delta \neq 0)$ there exists the so-called strip mode of the E_z^+/H_z^- family, with the constant of propagation given by

$$H_{T_0} = kU_0 \sqrt{\frac{\varepsilon\,[\ln(\xi/a) - \ln\delta_1]}{V_0^2\,[\ln(\xi/a) - \varepsilon\ln\delta_1]}}\ ; \qquad \delta = \sin(\delta/2) \to 0\ ;\ \ U_0^2 = (\varepsilon+1)/2\ . \tag{14}$$

In a similar way, for any narrow slot $(\theta \neq 0)$ there exists a so-called slot mode of the E_z^-/H_z^+ family, with the constant of propagation given by

$$h_{H_{00}} = k\left[U_0^2 + (kb)^{-2}(1 - a^2/b^2)^{-1}\ln^{-1}\theta_1\right]^{1/2}\ ; \qquad \theta_1 = \sin(\theta/2) \to 0\ . \tag{15}$$

As $\delta \to 0$, the strip mode field has $|H_z| \ll |E_z| \ll |H_t|, |E_t|$ showing the features of both quasi-E and quasi-T type modes. It is also a principal mode without low-frequency cutoff. In contrast, the slot mode is of the quasi-H type, as $\theta \to 0$, since it has $|E_z| \ll$

$|H_z|$, $|H_T|$, $|E_t|$ and extremely low but finite cutoff frequency tending to zero.

Fig. 2 presents qualitative pictures of cross-sectional E-fields of all the modes mentioned above. Both strip and slot modes are singular, since the former disappears from the spectrum if $\delta = 0$, while the latter, if $\theta = 0$. All the other modes transform, in a continuous way, from the modes of a Goubau line to the modes of a coaxial cable and its external domain.

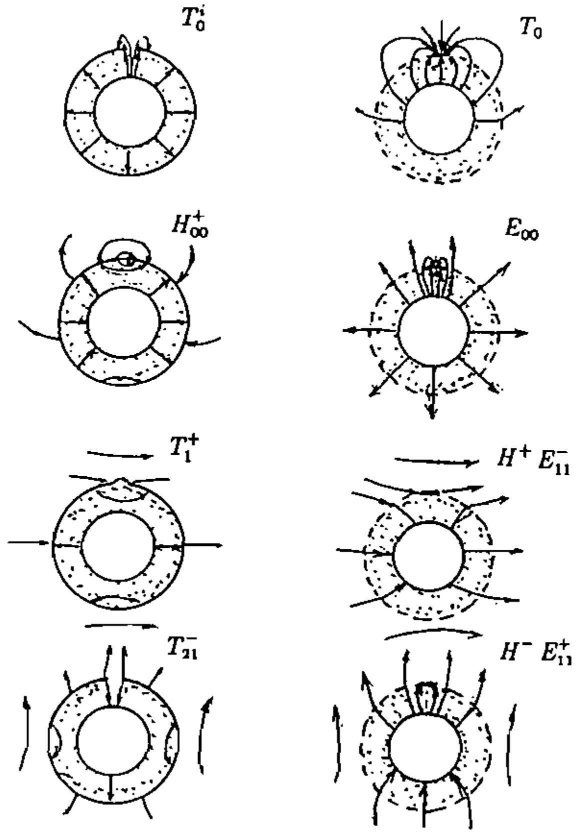

FIGURE 2: Mode field lines

4. NUMERICAL RESULTS

Here we present the results of computations of an iterative algorithm based on Newton's principle, applied to find the spectral points h_j, approximated by the characteristic values of the matrix $I - A_{mn}^{ij}(h)$, with $i,j = 1,2$ and $|m|$, $|n| \leq N_t$.

Fig. 3 shows the mode dispersion curves for a strip line with substrate parameters $R = a/b = 0.5$, $\varepsilon = 2.25$, and a narrow strip of angular width $\delta = 1°$. The singular quasi-T_0 mode (strip mode) dominates the whole range, especially in the low-frequency limit. The wavenumbers of other principal modes have different behavior in the quasistatic region, but all of them are only slightly perturbed by the strip in comparison to Goubau line modes (indicated by dashed lines). The perturbation is greater for the E_z^+/H_z^- family, being of the order of $0(\ln^{-1}\delta)$ as $\delta \to 0$, while the order of the E_z^-/H_z^+ mode family perturbation is $0(\delta^2)$. If the frequency of operation is higher than $ka = 2$, then all four principal waves are competitive, even though the first higher order modes, HE_{21}, are still far below slow-wave cutoff. Such a line evidently loses its quasi-single mode behavior, and strong coupling has to take place between modes when scattering from inhomogeneities is present.

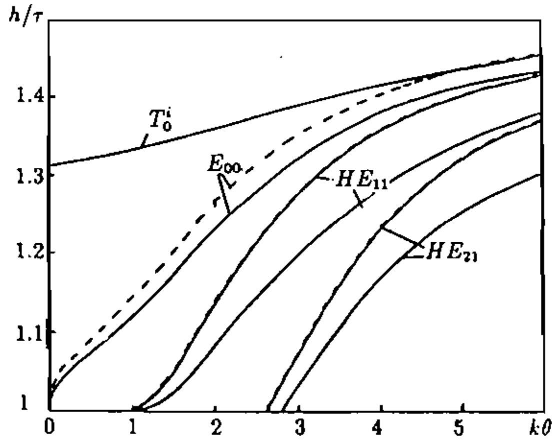

FIGURE 3: Mode dispersion curves. $\epsilon = 2.25$, $a/b = 0.5$, — : $\delta = 1°$, - - - : $\delta = 0°$.

When the substrate thickness parameter R is smaller, as in Fig. 4, the range of the strip mode dominating its behavior increases. In this case, we took the same line parameters, i.e. $R = 0.9$, $\epsilon = 2.32$ and $\delta = 0.05, 0.15$, as in [3], and recalculated the wavenumber into the effective dielectric constant $\epsilon_{eff} = (h/k)^2$. This gave us the possibility of comparing our approach with those of [3] and [4]. It turned out that, for the given strip widths (rather narrow ones), both approaches are in excellent agreement, differing by no more than 1% in the whole frequency range concerned. Wavenumbers of all the other principal modes are much closer to unity, but the Goubau line mode E_{00} becomes competitive for $ka \geq 10$. Note that only the dominant T_0 mode is sensitive to the strip-width shift from $\delta = 0.05$ to $\delta = 0.15$ (the dashed line in Fig. 4).

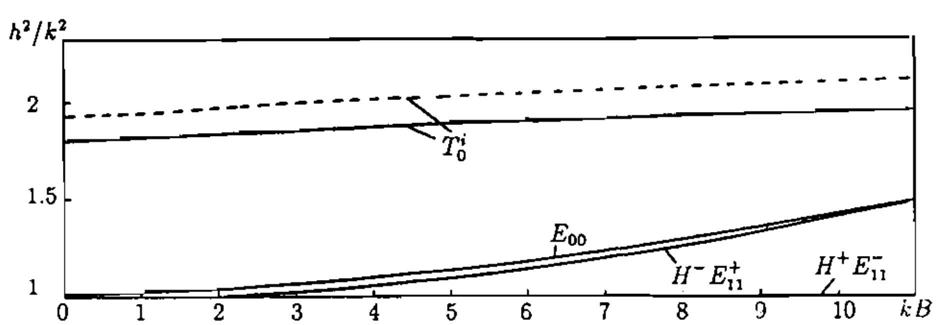

FIGURE 4: Mode dispersion curves. $a/b = 0.9$, $\epsilon = 2.32$. — : $\delta = 0.05$ rad; - - - : $\delta = 0.15$ rad.

Figures 5 and 6 show analogous curves for a slot line on the same cylindrical substrate for the two substrate parameters $R = 0.5$ and 0.9, respectively. As the slot width is small, they are similar to the same dependencies of coaxial cable modes (dashed lines). The only exception is the singular quasi-H_{00} mode (slot mode), which dominates at low frequencies until it remains above cutoff. Note that the slot mode is less sensitive to the inner loading of the guide than the other modes, because its nature is closely related to the slot itself. As has already been pointed out, the slot mode is not a principal one. Below cutoff, it turns into a leaky-wave mode in just the same way as all the other higher order modes. In the quasi-static limit, only the principal modes — quasi-T_0 and quasi-T_1^+ — are propagating losslessly, but their phase constants differ from unity by less that 0.001% in the whole range under consideration. The leakage constants of higher order modes below cutoff rise slowly until they equal the phase constants, and then much more rapidly with decreasing frequency.

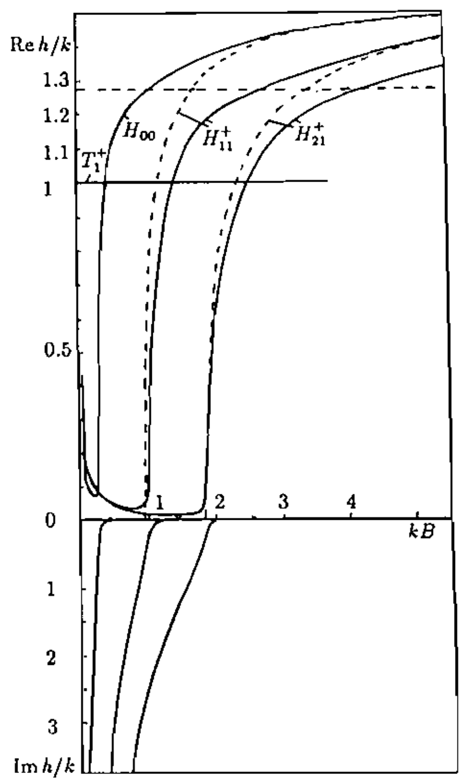

FIGURE 5: $a/b = 0.5$, $\varepsilon = 2.25$, — : $\theta = 1°$, - - - - : $\theta = 0°$.

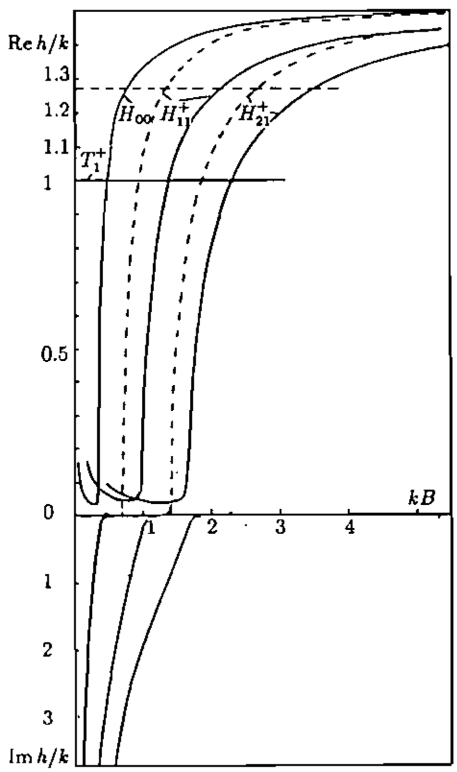

FIGURE 6: $a/b = 0.9$, $\varepsilon = 2.25$, — : $\theta = 1°$, - - - - : $\theta = 0°$.

Perhaps the most interesting results are presented in Fig. 7, which shows the transformation of slot line modes into strip line modes as the slot is widened while the strip is narrowed. The phase constants of all the slot-line modes decrease either monotonically, or after a small local increase. Those which come close to unity demonstrate a sharp change of dispersive behavior if the strip is narrowed further (compare with Fig. 4). This indicates a coupling (within the same mode family) with one of the principal modes of the guide that is much less sensitive to the strip/slot width relation. The manner of this coupling is the same as for modes in a partially screened circular dielectric core [5], [6]. If the strip is narrowed further, the principal wave transforms into a leaky one, strongly radiating into the surrounding free space. The TEM wave of the coaxial cable transforms into a strip mode of the cylindrical strip line, with the wavenumber reducing from $\varepsilon^{1/2}$ to $((\varepsilon+1)/2)^{1/2}$ in a monotonic manner.

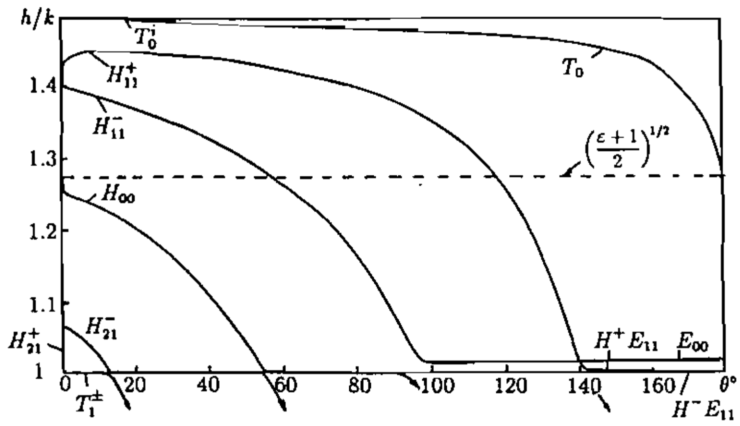

FIGURE 7: $\varepsilon = 2.25$, $kb = 2$, $a/b = 0.4$.

5. CONCLUSIONS

A mathematically correct full-wave method has been developed for the effective computation of mode characteristics in a partially screened circular dielectric core. This approach is applied to the simulation of microstrip and slot lines on a coated cylinder. The initial mode spectral problem is reduced to a homogeneous system of algebraic equations, with an operator of the Fredholm system, second kind. This enabled us to construct a highly effective numerical algorithm and check the validity of the approach, based on the dynamic Green's function formulation of a circular cylindrical substrate [3], [4]. The latter turned out to be quite accurate for the tested cases of narrow strips and low and middle frequencies. Also, wavenumbers of all other principal modes, as well as some higher order modes, have been computed, and the transformation of modes between circular microstrip lines and analogous slot lines has been investigated.

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