# ELECTROMAGNETIC WAVE SCATTERING BY A GRAPHENE-SANDWICHED THIN DIELECTRIC DISK ANALYZED USING THE GENERALIZED BOUNDARY CONDITIONS

Mikhail V. Balaban[1], Ana Vukovic[2], and Trevor M. Benson[2]

[1]Lab. of Micro and Nano-Optics, Institute of Radiophysics and Electronics NASU, Kharkiv 61085, Ukraine
[2]George Green Institute for Electromagnetics Research, University of Nottingham, NG7 2RD, Nottingham, UK
mikhail.balaban@gmail.com

## INTRODUCTION

Graphene is a planar one-atom-thick structured layer of carbon atoms which are arranged in a regular hexagonal pattern. Recently it has attracted the attention of the research community due to its amazing mechanical, electronic and optical properties. Graphene is a semi-metal or zero bandgap semiconductor with conductivity, which can be tuned either by electrostatic or magnetostatic gating. Graphene can also support surface plasmon waves and, if patterned, resonance modes at low-THz frequencies. These properties make it a very promising material for the development of ultrathin fast nanoelectronic devices [1]. Thin dielectric disks attract a significant interest in the millimetre-wave and terahertz ranges and in micro and nano-optics. This is due largely to the use of semiconductor, crystal and polymeric thin flat microdisks as laser resonators with injection-type or photo-pumped active zones. The working frequencies of these devices are at terahertz and optical wavelengths [2,3]. The present work is aimed at developing a mathematical model for electromagnetic wave scattering by a thin graphene-dielectric-graphene sandwich-like disk. The model is used to study the influence of graphene layers on the resonance frequencies of a thin dielectric disk.

To study the electromagnetic wave scattering by a thin dielectric disk covered by graphene it is necessary to couple the Maxwell boundary value problem for a thin disk with a phenomenological model of graphene conductivity. One of the main challenges of such an approach is to involve the near-zero thickness of the graphene cover into the model. Another difficulty is to find a numerically effective and stable method to solve this boundary problem. We propose to couple the equivalent resistive boundary conditions as a model of each graphene cover with two-side generalized boundary conditions (GBS) as a model of a thin dielectric disk and then obtain effective GBS for a sandwich-like disk. To reduce the problem to a matrix equation we follow the method of analytical regularization developed for a thin dielectric disk scattering problem in [4].

## PROBLEM STATEMENT

We consider the scattering of a given time-harmonic electromagnetic wave by a thin graphene-sandwiched dielectric disk (Fig. 1). Assume that the thickness of the disk is $\tau$ and its radius is $a$. Introduce cylindrical dimensionless coordinates $(\rho = r/a, \varphi, \zeta = z/a)$ with the origin at the disk centre. Decompose the total field as a sum of the incident and the scattered fields. Assume that the scattered field satisfies homogeneous Maxwell equations outside the disk and the total field satisfies the following effective GBC at the median section of the disk $(\rho < 1)$:

$$Z_0\left(H_{tg}^{+} - H_{tg}^{-}\right) = -1/2\, R_{eff}^{-1} \vec{n} \times \left(E_{tg}^{+} + E_{tg}^{-}\right) \quad (1)$$

$$\left(E_{tg}^{+} - E_{tg}^{-}\right) = 1/2\, Q_{eff}^{-1} \cdot Z_0\, \vec{n} \times \left(H_{tg}^{+} + H_{tg}^{-}\right) \quad (2)$$

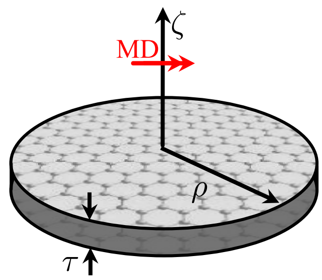

Figure 1. Magnetic dipole above a sandwich-like disk

Here $Z_0$ is the free-space impedance, $\vec{n}$ is the unit vector normal to the disk surface, $R_{eff}$ and $Q_{eff}$ are effective electrical and magnetic resistivities, which depend on electrical properties of the dielectric material, graphene conductivity, frequency of the incident field, and sandwiched disk thickness. Expressions for the effective resistivities will be obtained in the next paragraph. For completeness, assume that the total field satisfies the radiation condition and the condition of local integrability of power. Also assume that the scattered field is continuous everywhere outside the disk. To apply the method of analytical regularization developed for a thin dielectric disk scattering problem it is necessary to supplement the GBC (1), (2) by the condition of the absence of equivalent currents out of disk's surface in the plane of its median section $(\zeta = 0, \rho > 1)$.

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GENERALIZED BOUNDARY CONDITIONS FOR A DIELECTRIC LAYER COVERED BY GRAPHENE

Now we will obtain the two-side GBC for the modelling of a graphene-dielectric-graphene sandwich-like infinite layer. At first, split the whole sandwich into three single layers (Fig. 2) and suppose there are non-zero free space gaps between each of layers. Define $(E_{tg}^{1,+}, H_{tg}^{1,+})$ and $(E_{tg}^{1,-}, H_{tg}^{1,-})$ as the limits of the field components tangential to layer at the top and the bottom sides of the top layer, respectively. Also define $(E_{tg}^{2,+}, H_{tg}^{2,+})$ and $(E_{tg}^{2,-}, H_{tg}^{2,-})$ as the limits of the tangential field components at the top and the bottom surfaces of the dielectric layer. Finally, define $(E_{tg}^{3,+}, H_{tg}^{3,+})$ and $(E_{tg}^{3,-}, H_{tg}^{3,-})$ as the limit values of the field components at the bottom graphene layer top and bottom sides, respectively.

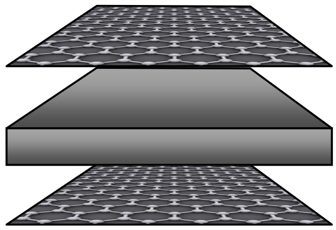


Figure 2. Split layers

Consider the following boundary conditions:

$$\left(E_{tg}^{1,+} + E_{tg}^{1,-}\right) = 2\sigma_{Gr}^{-1} \cdot \vec{n} \times \left(H_{tg}^{1,+} - H_{tg}^{1,-}\right), \qquad \left(E_{tg}^{1,+} - E_{tg}^{1,-}\right) = \vec{0}, \tag{3}$$

$$\left(E_{tg}^{2,+} + E_{tg}^{2,-}\right) = 2Z_0\, R_{Diel} \cdot \vec{n} \times \left(H_{tg}^{2,+} - H_{tg}^{2,-}\right), \qquad \left(E_{tg}^{2,+} - E_{tg}^{2,-}\right) = Z_0/2\, Q_{Diel}^{-1} \cdot \vec{n} \times \left(H_{tg}^{2,+} + H_{tg}^{2,-}\right), \tag{4}$$

$$\left(E_{tg}^{3,+} + E_{tg}^{3,-}\right) = 2\sigma_{Gr}^{-1} \cdot \vec{n} \times \left(H_{tg}^{3,+} - H_{tg}^{3,-}\right), \qquad \left(E_{tg}^{3,+} - E_{tg}^{3,-}\right) = \vec{0}. \tag{5}$$

They are conditions for the electromagnetic field on a graphene layer (3), (5) and two-side GBC for a thin dielectric layer (4). Here, $R_{diel}$ and $Q_{diel}$ are the electric and magnetic resistivities given as $R_{diel} = iZ/2\cot(\sqrt{\varepsilon_r \mu_r} k\tau/2)$, $Q_{diel} = R/Z^2$ if $k\tau \ll 1$ and $|\varepsilon_r \mu_r| \gg 1$, $Z_0$ is the free-space impedance, $Z$ is the relative impedance of the disk material, $k = \omega / c$ is the wavenumber, $\varepsilon_r$ and $\mu_r$ are the relative permittivity and permeability, respectively. Besides, $\sigma_{Gr}$ is the graphene surface conductivity, which can be determined from the Kubo formalism and expressed as a sum of intraband $\sigma_{intra}$ and interband $\sigma_{inter}$ contributions given by the following expressions [5]:

$$\sigma_{intra} = \frac{i e^2 k_B T}{\pi\hbar(\omega + i/t_{relax})}\left(\frac{\mu_c}{k_B T} + 2\ln\left[\exp\left(-\frac{\mu_c}{k_B T}\right) + 1\right]\right), \tag{6}$$

$$\sigma_{inter} \approx \frac{1}{4\hbar} e^2 \left(f_d\left[-\hbar/2(\omega + i/t_{relax})\right] - f_d\left[\hbar/2(\omega + i/t_{relax})\right]\right), \tag{7}$$

where $e$ is the charge of an electron, $k_B$ is the Boltzmann constant, $T$ is temperature, $\hbar$ is the reduced Planck constant, $t_{relax}$ is the relaxation time, $\mu_c$ is the chemical potential, and $f_d$ is Fermi-Dirac distribution function.

Now assume that the thicknesses of each gap are equal to zero. Then $(E_{tg}^{1,-}, H_{tg}^{1,-}) = (E_{tg}^{2,+}, H_{tg}^{2,+})$ and $(E_{tg}^{2,-}, H_{tg}^{2,-}) = (E_{tg}^{3,+}, H_{tg}^{3,+})$ and after some algebraic transformations we obtain the following:

$$\left(E_{tg}^{1,+} + E_{tg}^{3,-}\right) = 2Z_0\, R_{GrDielGr} \cdot \vec{n} \times \left(H_{tg}^{1,+} - H_{tg}^{3,-}\right), \tag{8}$$

$$\left(E_{tg}^{1,+} - E_{tg}^{3,-}\right) = Z_0/2\, Q_{GrDielGr}^{-1} \cdot \vec{n} \times \left(H_{tg}^{1,+} + H_{tg}^{3,-}\right), \tag{9}$$

where

$$R_{GrDielGr} = R_{Diel} / \left(2Z_0\sigma_{Gr}R_{diel} + 1\right), \; Q_{GrDielGr} = Z_0\sigma_{Gr}/2 + Q_{Diel} \tag{10}$$

Thus we have obtained the two-side GBCs for the field components tangential to the layer. They can be used for the modelling of a graphene-dielectric-graphene sandwich layer. The use of these two-side GBCs enables one to exclude consideration of the field inside the sandwich while solving the scattering problem.

By a similar but reverse procedure, following [6], one can modify the two-side GBCs obtained so that the sandwiched layer can be reduced to a boundary of zero thickness. Finally, that leads to the effective GBC (1), (2) at the median section of the layer with effective resistivities given by the following expressions:

$$R_{eff} = \frac{4Q_0R_0R_{GrDielGr} + R_{GrDielGr} - 2R_0}{4Q_0R_0 - 8Q_0R_{GrDielGr} + 1}, \qquad Q_{eff} = \frac{4R_0Q_0Q_{GrDielGr} + Q_{GrDielGr} - 2Q_0}{4R_0Q_0 - 8R_0Q_{GrDielGr} + 1}, \tag{11}$$

where $R_0 = Q_0 = i\cot\left(k\tau/4\right)/2$ and $R_{GrDielGr}$, $Q_{GrDielGr}$ are given by (10).

## NUMERICAL RESULTS AND DISCUSSION

For the numerical study of the electromagnetic field scattering by the sandwich-like disk we consider the horizontal magnetic dipole as a source of the incident field. The dipole is located on the disk axis and distance $\zeta_{in}$ above it. We consider the sandwich-like disk of Fig. 1 with the radius 50 microns and take the dielectric constant $\varepsilon_r = 60 + i \cdot 0.0006$. The graphene conductivity is taken for room temperature, 1 ps electron relaxation time and several values of the chemical potential. Fig. 3 shows the normalized radiation power of the dipole in the presence of the sandwiched disk as a function of the frequency in terahertz range. One can see the resonance behaviour of the curves. The first low-frequency resonance on each curve (except the black one) corresponds to the surface plasmon mode of the graphene disk. The next resonances correspond to the dielectric disk modes of the “dipole” type, i.e. with several field variations along the radius and one variation along the azimuth. One can see a strong shift of the plasmon resonance frequency while changing chemical potential of the grapheme (this is the same as electrostatic gating.) This effect can be used for the tuning of the resonance frequency of the sandwich-like disk in the applications related to the sensing of refractive index of the host medium.

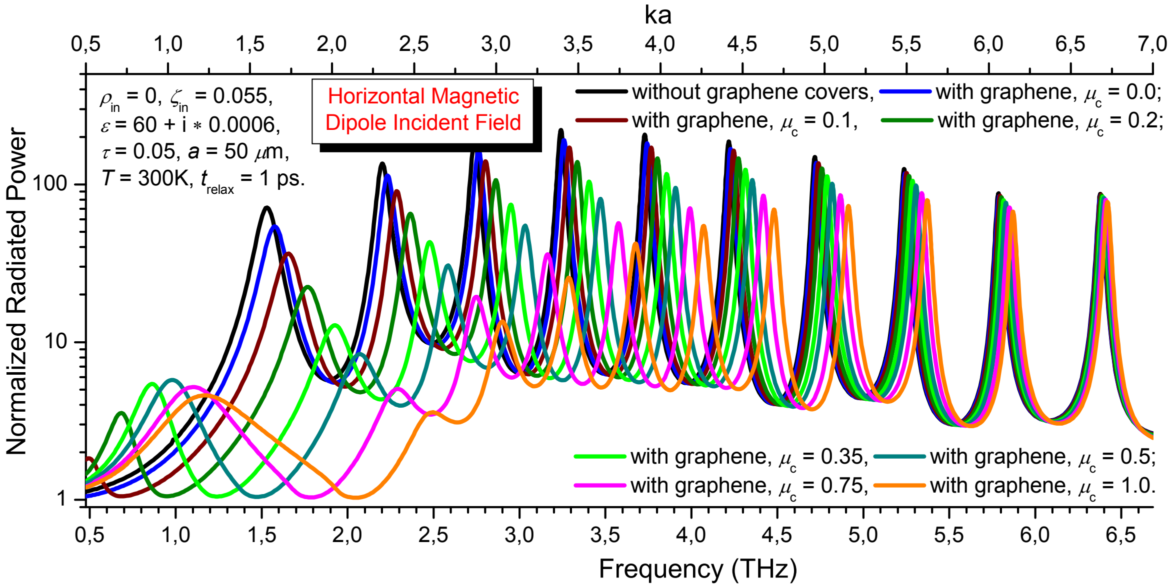


Figure 3. Normalized radiation power of the on-axis horizontal magnetic dipole in the presence of a sandwich-like disk

## ACKNOWLEDGMENTS

This work was partially supported by the National Academy of Sciences of Ukraine via the State Target Program “Nanotechnologies and Nanomaterials” and the European Science Foundation via the Research Networking Programme “Newfocus” in the form of exchange travel grant to the first author.

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