Modification of Spontaneous Radiation in the Presence of a 3-D Thin Dielectric Microdisk and Partial Justification of the 2-D Effective Index Model

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ABSTRACT
The spontaneous emission of a molecular dipole in the presence of a thin dielectric microdisk is studied as a 3D solution of Maxwell's equations with two-sided generalized boundary conditions, local energy finiteness, and a radiation condition at infinity. Results show the radiative and non-radiative decay rates display resonance maxima associated with the disk natural frequencies which can be well explained through the effective- refractive-index approximation.

Keywords: dielectric disk, generalized boundary conditions, analytical regularization, dual integral equations

1. INTRODUCTION

The term "Purcell effect" comes from quantum optics and labels resonant enhancement or, in more a general sense, modification of the spontaneous emission of an atomic or molecular dipole in non-homogeneous environment. Today's interest, in optics, in this phenomenon is explained by the ability of various nano- and micro-size particles to increase spontaneous emission by many orders of magnitude; such enhancement leads directly to applications connected to microlasers and cavity quantum electrodynamics [1-3]. More precisely the study of this effect deals with the radiated and absorbed powers associated with elementary dipoles near various resonant objects. As a rule the Purcell effect has been estimated using the so-called "Purcell factor" which is proportional to the ratio of the resonant mode quality factor to the mode volume. However, it has been recently convincingly argued that this factor, originally derived for closed cavities with imperfectly conducting walls, cannot be used in the case of open resonators such as dielectric or semiconductor microdisks. The reasons are twofold: here the natural modes do not form a complete orthogonal set of field functions and the resonance does not lead to one-term representation of the spontaneous emission rate. Therefore for an accurate estimation of the Purcell effect for open resonators it is mandatory to use full-wave modelling methods and convergent computational techniques.

2. FORMULATION

We consider the problem of finding the electromagnetic field emitted by an elementary electric dipole (EED) parallel to a thin dielectric disk and located as depicted in Fig. 1.

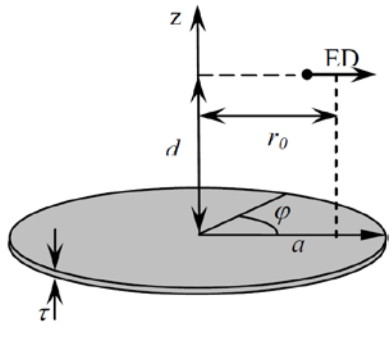

Fig. 1. Geometry of microdisk

If the disk thickness is small, as typical for microcavity lasers, one can neglect the field inside the disk and shrink its volume to the median section, at the expense of introduction of two-side generalized boundary conditions (GBC). We expand the field into Fourier series in the azimuth coordinate and use scalar and vector Hankel integral transforms in the radial coordinate. Substituting these functions into GBC, we obtain a set of dual IEs for each azimuth order *m*. Then we use the method of analytical regularization to invert IE static parts and reduce them to two pairs of coupled Fredholm second kind IEs with smooth kernels [4]. The features of the latter IEs guarantee the convergence of numerical solution if the order of discretization is increased.

3. NUMERICAL RESULTS

As computations have shown, the radiated and absorbed powers display resonances which correspond to the disk eigenmodes of different azimuth orders (see Fig. 2). Note that the Q-factors of these resonances become higher with increasing the azimuth order as typical to the whispering gallery modes (WGM). The resonances revealed in the normalized powers can be explained using the effective refractive index model of the disk. They are caused by the standing waves formed due to the reflections of the guided wave of the dielectric slab at the disk rim. This observation can serve as a *de-facto* justification of the empirical 2-D model of disk cavity with effective refractive index, from the viewpoint of rigorous Maxwell theory.

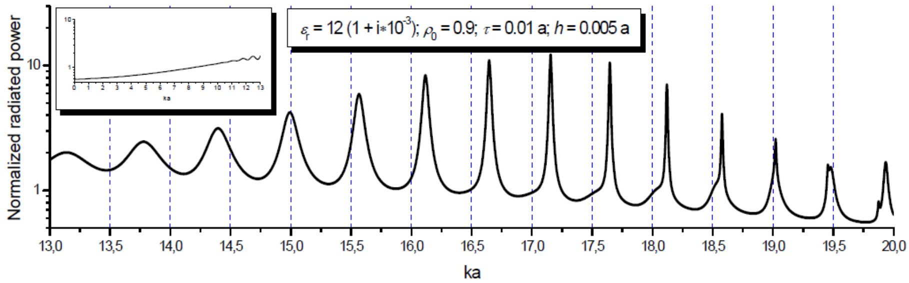

Fig. 2. Normalized radiated power for off-axis EED in the disk presence versus frequency ka.

The 3-D far field pattern corresponding to one of the maxima on the plot in Fig. 2 is presented in Fig. 3.

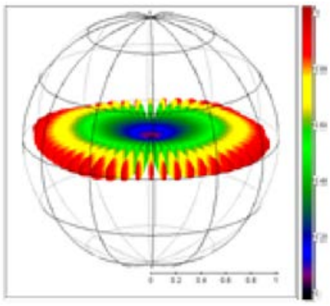

Fig. 3. Total far field pattern for the same disk and dipole as in Fig. 2 with ka = 19.4544318

This in-resonance pattern clearly displays two features: first, the radiation is concentrated in the disk plane and, second, in that plane the pattern shows 50 almost identical lobes characteristic for WGM with $m = 25$.

4. CONCLUSIONS

We have accurately quantified the Purcell effect or, equivalently, the modification of powers radiated and absorbed due to horizontal EED in the presence of a thin dielectric microdisk, by studying it from first principles. This has been done over a wide range of the normalized wavelengths. Thus, the analytical-numerical method which has been applied here places thin material disks in the same position as spherical scatterers in the sense that they can be computed very economically and with controlled accuracy.

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