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# Periodicity Matters

## *Grating or lattice resonances in the scattering by sparse arrays of subwavelength strips and wires.*

This article reviews the nature and history of the discovery of high-quality natural modes existing on periodic arrays of many subwavelength scatterers; such arrays can be viewed as specific periodically structured open resonators. These grating modes (GMs), like any other natural modes, give rise to the associated resonances in electromagnetic-wave scattering and absorption. Their complex wavelengths are always located very close to (but not exactly at) the well-known Rayleigh anomalies (RAs), determined only by the period and the angle of incidence. This circumstance has long been a reason for their misinterpretation as RAs, especially in the measurements and simulations using low-resolution methods. In the frequency scans of the reflectance or transmittance, GM resonances usually develop as asymmetric Fano-shape spikes. In the optical range, if a grating is made of subwavelength-size noble-metal elements, then GMs exist together with better-known localized surface-plasmon (LSP) modes. Thanks to high tunability and considerably higher *Q*-factors, the GM resonances can potentially replace the LSP-mode resonances in the design of nanosensors, nanoantennas, and solar-cell nanoabsorbers.

## INTRODUCTION

Although GMs can be found in various large or infinite arrays made of subwavelength metallic and dielectric elements, and in all wavelength ranges, we will concentrate our discussion around infinite arrays of circular silver wires and thin silver strips (Figure 1) in the optical range. As illustrative material, we will use the data from [1] and [2].

It should be noted that some remarkable circumstances appear in the analysis of time-harmonic ($\sim e^{-i\omega t}$) electromagnetic scattering in the optical range. This range stretches between the wavelengths of 300 and 900 nm and presents specific features when it comes to full-wave modeling. These all relate to the description of material properties of scatterers and are absent in the microwave range.

- The concept of the perfect electric conductor (PEC) must be discarded because even good metals are sizably lossy, although good dielectrics can still be assumed lossless.
- Both real and imaginary parts of the dielectric permittivities of metals are the functions of the wavelength, and only some dielectrics show almost constant permittivities.



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- The real parts of the dielectric functions of metals display plasma-like negative values in the optical range [3], thus the term *negative dielectrics* has been coined.
- The imaginary parts of the dielectric permittivities of semiconductors can be negative, i.e., correspond to the material gain (or negative loss), under the pumping.

These circumstances mean that the conventional microwave PEC boundary conditions are not valid for metals and either dielectric-interface or surface-impedance conditions must be used. Negative-dielectric or inductive-impedance behaviors of metals in the optical range has important consequences in the appearance of optical LSP modes on subwavelength, e.g., nanosize particles. Noble-metal nanowires are also known to display intensive optical LSP resonances in the visible range if illuminated with the H-polarized (i.e., polarized orthogonally to the wire axis) light [4].

## INDIVIDUAL STRIPS AND WIRES IN OPTICAL RANGE

The optical LSP resonance wavelengths depend primarily on the shape of the nanoscatterer or its cross section [4], [5]. For instance, a thin, flat metal nanostrip of the relative dielectric permittivity $\mathrm{Re}\,\varepsilon_{\mathrm{met}} < 0$ in a host medium with $\mathrm{Re}\,\varepsilon_h > 0$ shows a broad peak in the scattering and absorption cross sections. The resonance wavelength $\lambda^P$ is determined by the standing-wave condition of the short-range guided surface-plasmon (SP) wave of the corresponding infinite metal slab bouncing between the strip edges [6], [7]. This condition demands that the strip width $w$ equals half of the SP wavelength. That value depends on the slab thickness and frequency, so that for a $150 \times 50\ \mathrm{nm}^2$ silver strip in free space $\lambda^P \approx 410$ nm and for a $150 \times 20\ \mathrm{nm}^2$ strip, it shifts to 550 nm (see [7, Fig. 12]). The higher-order LSP conditions correspond to $n = 2, 3, \ldots$ half wavelengths of the SP wave on the strip width and, hence, are observed at smaller $\lambda$.

A thin-circular metal nanowire has a single broad peak in the optical scattering and absorption cross sections slightly above the wavelength $\lambda^P$, where $\mathrm{Re}\,\varepsilon_{\mathrm{met}}(\lambda^P) = -\mathrm{Re}\,\varepsilon_h$. For a silver wire in free space, $\lambda^P \approx 338$ nm [4]. This follows from the analytical study of the plane-wave scattering by a wire using the separation of variables that can be further simplified using the small-argument asymptotics of cylindrical functions. This study shows that the circular wire possesses an infinite number of closely spaced double-degenerate LSP eigenmodes of the azimuth orders $n = 1, 2, \ldots$. They appear as complex poles of the field as a function of the wavelength with an accumulation point at a complex location near to $\lambda^P$. They have slightly different $n$-dependent real parts shifted to the larger $\lambda$ from 338 nm and imaginary parts that get smaller for larger $n$ because of smaller losses. However, in the scattering, the corresponding resonance peaks overlap because the losses are considerable, although the largest contribution comes from the dipole terms with $n = 1$. Noncircular-wire scattering analysis needs more elaborated techniques such as volume or boundary integral equations [4]–[7]. They also reveal shape-dependent LSP modes of different types and symmetries.

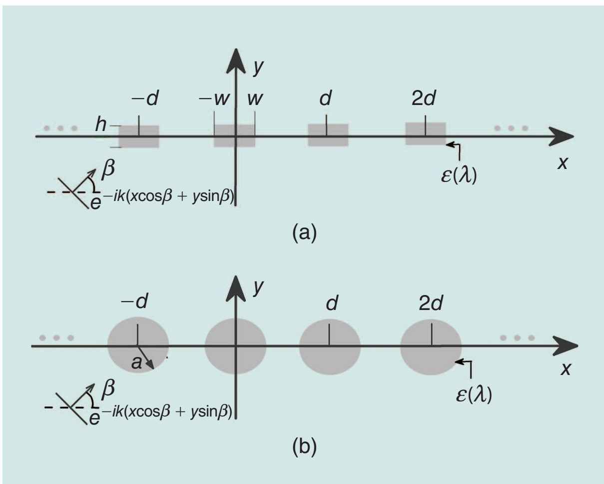


**FIGURE 1.** The cross-sectional geometries of the infinite gratings made of (a) flat strips and (b) circular wires illuminated by a plane wave.

**The optical GM resonances have mostly been studied, both theoretically and experimentally, on chains and gratings of 3-D particles.**

In the scattering, optical LSP resonances are the signatures of the underlying LSP poles. If the shape of the wire cross section is fixed, their wavelengths are specific for every host medium that makes the sensing of the medium refractive index possible by means of measuring the LSP peak wavelength [8]. The accurate reading of the primary (largest $\lambda$) LSP resonances is spoiled by their comparatively low $Q$-factors, which are dominated by the value $|\mathrm{Re}\,\varepsilon_{\mathrm{met}}/2\,\mathrm{Im}\,\varepsilon_{\mathrm{met}}|$ that is only 40 at the infrared edge of the visible range.

Although the optical properties of the LSP modes of pairs (dimers) or small clusters of coupled metal wires or strips have already been well documented (see [9] and the references therein), large periodic ensembles of them (e.g., chains, arrays, and gratings) remain less studied. As mentioned in the "Introduction" section, such ensembles display the existence of the other periodicity-caused GM resonances whose nature is still not always correctly understood. In the following, we present a brief narrative of related publications and discuss the remarkable properties of these non-LSP optical resonances found in nanogratings of circular silver wires and thin strips (Figure 1). For simplicity, the gratings are assumed to be infinite and stand in free space.

It should be added that the optical GM resonances have mostly been studied, both theoretically and experimentally, on chains and gratings of three-dimensional (3-D) particles (see [10]–[23]). The intrigue surrounding the optical GM resonances on various nanogratings of metal scatterers consists in the fact that, in the early studies, they were frequently mixed up with more conventional optical LSP resonances, although mysteriously coupled to RAs. The failure to recognize their differing natures can be seen in the use of plasmon-related terminology such as *radiatively nondecaying plasmons, supernarrow plasmon resonances, subradiant lattice plasmons,* and *plasmon resonances based on diffraction coupling of localized plasmons* [10], [13]–[18]. This started changing recently and now the terms like *collective resonance* [19]–[21] and *photonic resonance* [22] seem to dominate, although GRs are sometimes still confused with RAs [23]. Note that when the GM resonances were found in the gratings of not metal but dielectric particles [11], [12], they did not obtain any specific name at all. This observation tells much about the embarrassment of early researchers about the nature of GMs. Still, the fact that the GMs and associated resonances exist on the gratings of both metallic and dielectric elements makes it clear that they are solely caused by the periodicity and are not exotic plasmons.

## FLAT INFINITE THIN-STRIP GRATINGS

Flat gratings made of thin noble-metal strips [see Figure 1(a)] have always been attractive both at microwave frequencies and in optics as easily manufactured components that are able to provide polarization discrimination [24], [25] (the so-called Hertz effect). The theoretical study of the scattering by strip gratings was initially done assuming their flat layout, free-space location, infinite extension, zero thickness, and PEC conditions [26]–[29]. Here, an important research instrument was introduced in [30]: Floquet expansion of the field function in terms of spatial harmonics, also called *diffraction orders*.

Under these assumptions, the reflection and transmittance spectra of flat infinite strip gratings show only the RAs at the passing-off wavelengths

$$\lambda_{\pm m}^{\mathrm{RA}} = (d\sqrt{\varepsilon_h}/m)(1 \mp \cos\beta), \quad m = 1, 2, \ldots, \tag{1}$$

so that in free space under the normal incidence $\lambda_{\pm m}^{\mathrm{RA}} = d/m$.

If an infinite flat PEC-strip grating is supported by a thin-dielectric substrate, then strong GM resonances appear [31] in its reflectance and transmittance. This point will be addressed in the "GM Resonances at Microwaves" section. Note that the LSP resonances are absent on PEC strips. However, a gold-strip grating lying on a dielectric substrate displays both LSP and GM resonances [32] in the optical range.

The GM resonances on the free-standing infinite non-PEC strip gratings were found first for thin dielectric strips in [33], although, in the H-wave case, the corresponding narrow peaks were missed because the grid of computation points, in frequency, was too coarse. This was clarified in the subsequent studies of impedance-strip [34] and silver-nanostrip [2] gratings. In [2], it was analytically shown that the complex wavelengths of GMs tend to $\lambda_m^{\mathrm{RA}}$ if the metal strip width or thickness gets smaller. Numerical study of both optical LSP and GM resonances on finite gratings of many silver strips in the flat and comb-like configurations have been published in [35]–[37].

To highlight the differences between the conventional LSP and GM resonances in the visible-light scattering by periodic noble-metal scatterers, we present some numerical data for an infinite grating of thin silver strips illuminated by a normally incident H-polarized plane wave of the unit amplitude. The dispersion of the complex dielectric permittivity of silver has been taken into account using the experimental data for the real and imaginary parts from [3].

The plots of reflectance, absorbance, and transmittance as a function of the wavelength are presented in Figure 2. They were computed by the advanced meshless analytical regularization code of [2], in which convergence is guaranteed. The method is based on the use of generalized boundary conditions on each strip [7] and analytical inversion, in the H-wave case, of hypertype singularity in associated equations. This leads to the final matrix equation of the Fredholm second-kind type. The silver-strip dimensions were taken as $150 \times 50$ nm$^2$, and the period varies from 350 to 800 nm. The plots demonstrate one broad LSP resonance of enhanced reflection and absorption at 410 nm, associated with the first-order standing-wave mode built on the short-range SP wave bouncing between the edges of each strip. In addition, one can see one or two much sharper GM resonances at the wavelengths $\lambda^G_{1,2}$ slightly larger than the period and half-period of the grating. These resonances, if well distanced from the LSP ones, display the asymmetric Fano shapes (double spikes). However, if the period $p$ coincides with that of the LSP mode, then the symmetric narrowband optically induced transparency effect is observed (see the curves for $d = 400$ and 800 nm).

In Figure 3(a), we demonstrate this effect in detail for the grating made of 10-nm thin silver strips. Such a reduced thickness is usual for today's nanotechnologies operating with electron-beam lithography and other techniques. Here, one can see two broad LSP resonances in the visible-light range at ~630 and 380 nm, associated with the first- and third-order optical LSP modes on each strip. One can also see an extremely sharp Fano-shape double spike at the wavelength slightly larger than the period [see the zoomed-in view in Figure 3(b)]. This is the effect of the GM resonance whose near-field is shown in Figure 3(c) and (d).

According to [2], at the normal incidence, the normalized frequencies $\kappa = d/\lambda$ of the GM poles for a grating of non-magnetic strips with thickness $h/\lambda \ll 1$ and width $w$ in the free space have the following asymptotic values:

$$\kappa^{G(E,H)}_{\pm m} = |m| - \delta_{|m|}, \quad (2)$$

$$\delta_{|m|} = (1/2)|m|^3 \chi^2_{E,H} (\pi w h)^2 d^{-4} + O(\chi^4_{E,H} w^4 h^4 d^{-8}), \quad (3)$$

where $\chi_E = \varepsilon - 1$, $\chi_H = 1$, and $m = 1, 2, \ldots$. This means that, unlike the optical LSP modes, the GM $Q$-factors tend to infinity if $h/d \to 0$ both for the lossy and lossless dielectric strips and for the metal strips in either polarization. Note that the imaginary part of $\kappa^G_{\pm m}$ is a small value that is asymptotically a square of the small deviation of its real part from the RA.

In fact, (2) has been symbolically presented in several earlier publications, e.g., see [11, eq. (19)]; however, the deviation $\delta_m$ of the pole from the RA branch point had never been derived for a strip grating before [2].

In the scattering problem, if the incident wave length approaches the real part of the $m$th natural GM wavelength, then the $m$th Floquet harmonic amplitude $a_m$ takes a large value proportional to the GM $Q$-factor $Q_{G(E,H)m}$. As visible from (2), $\mathrm{Re}\,\kappa^G_m < m$, and, hence, the $m$th harmonic exponentially decays in the normal to the grating direction. As a result, the peak value of $|a_m|$ in the GM resonance is not restricted by the power conservation law.

Under the normal incidence, the optical field near the grating is dominated by the intensive standing wave built of two identical Floquet harmonics with numbers $\pm m$ [2]. For the plots in Figure 3(c) and (d), $m = 1$ and, hence,

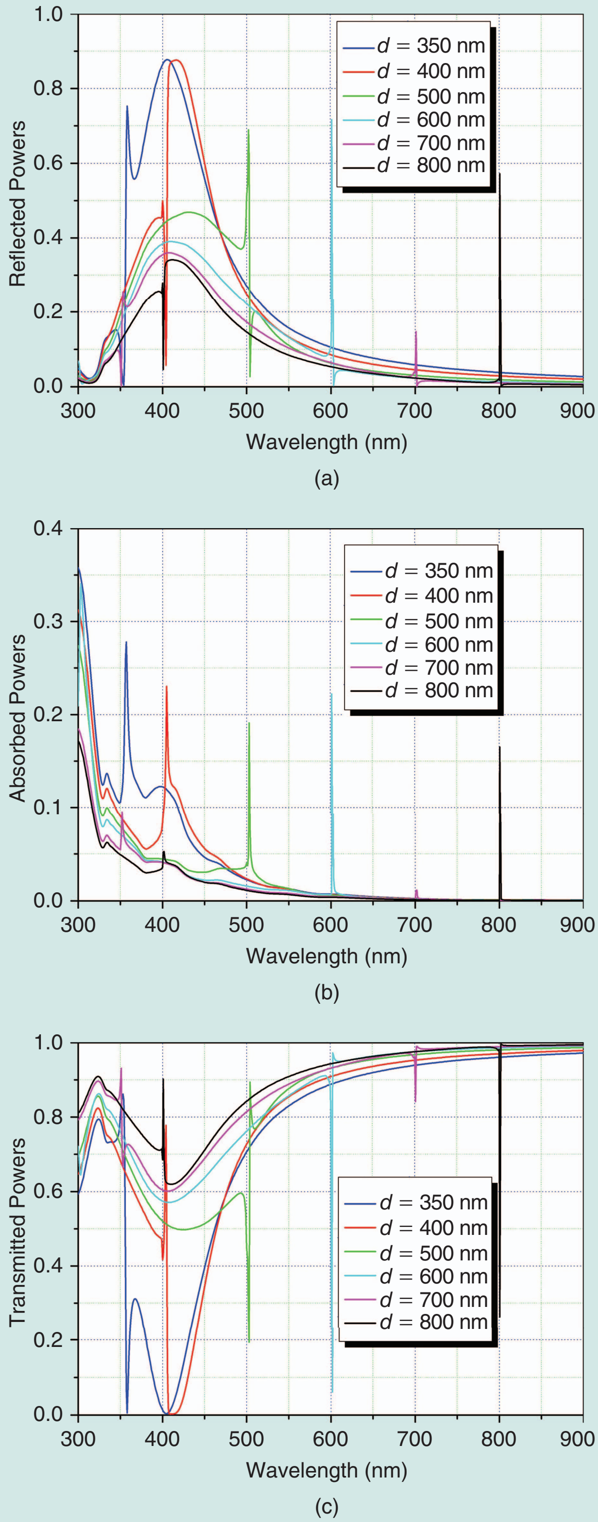


**FIGURE 2.** The wavelength dependences of (a) reflectance, (b) absorbance, and (c) transmittance powers of the different period gratings of thin silver strips with dimensions $150 \times 150$ nm$^2$, H-polarization, and normal incidence. (Figure reused with permission from [1].)

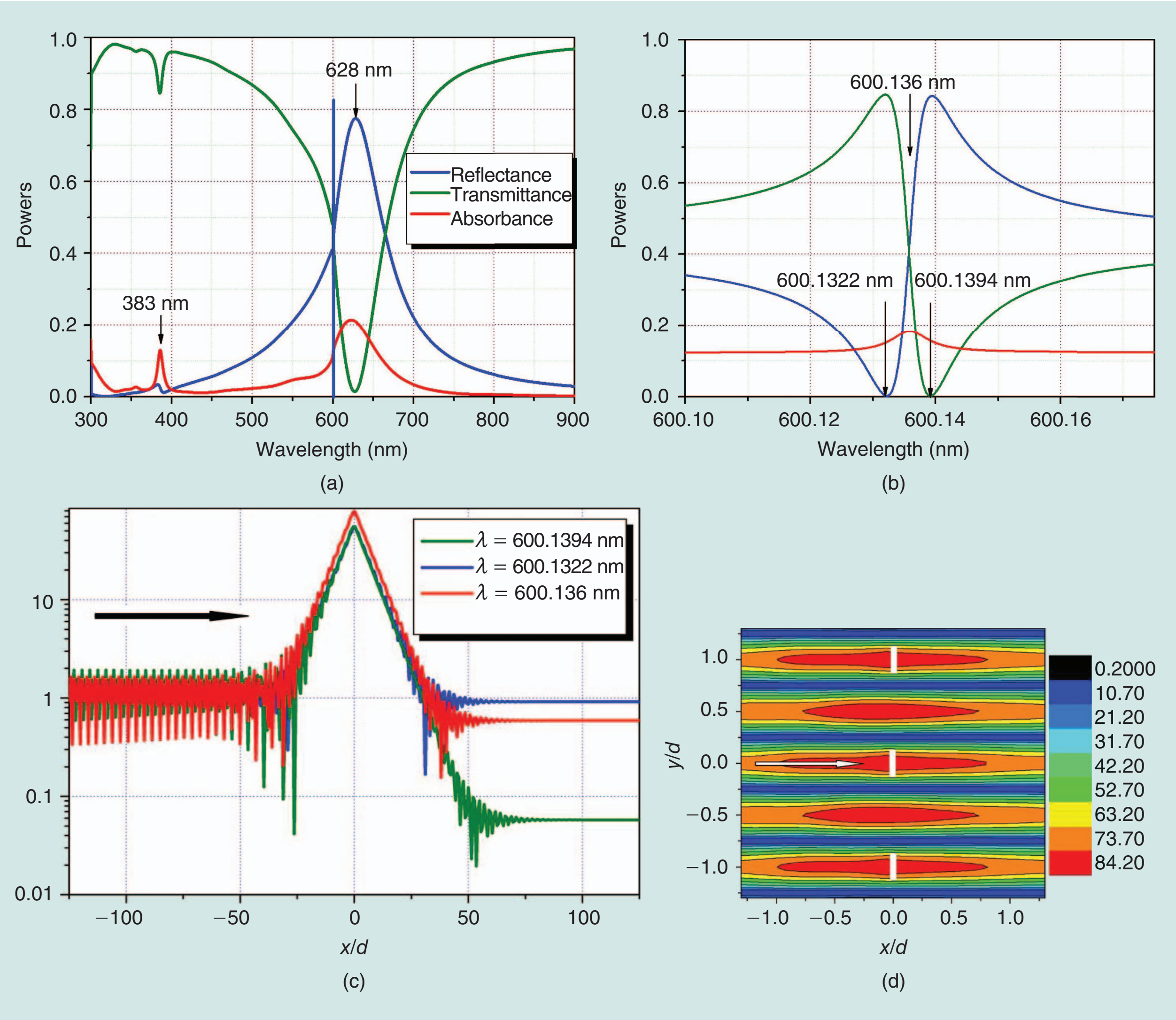


**FIGURE 3.** (a) The reflectance, transmittance, and absorbance as a function of the wavelength for the scattering of the normally incident H-wave from the grating of silver strips. $\varphi = 0^0$, $2w = 150$ nm, $p = 600$ nm, and $h = 10$ nm. (b) A zoomed-in view of (a) in the vicinity of GM resonance. (c) The magnetic field profile along the line $y = 0$ in the GM resonance. (d) The magnetic field pattern on three periods at $\lambda^G = 600.136$ nm. (Figure reused with permission from [2].)

$$H \approx 2a_1 e^{ik\sqrt{1-\frac{1}{(\kappa_1^{GH})^2}}|y|} \cos(2\pi x/d) \approx Q_{GH1} e^{-\frac{|y|}{dQ_{GH1}}} \cos(2\pi x/d). \quad (4)$$

This is fully consistent with the near-field patterns observed in Figure 3(c) and (d). Note that in the GM resonance, enhanced near-field stretches to approximately 50 periods on each side of the silver-strip grating, and the peak value is ~95. This is around 25 times larger than in the optical LSP resonance whose near-field bright spots are small and stick to the strips [2]. At the inclined incidence, the RAs split into pairs, $+m$ and $-m$, each accompanied by its GM pole.

In the case of finite silver-strip gratings, at least ten strips are needed to produce a GM-related peak [35], [36]. The GM near-field pattern is visible along the grating except a few periods near the ends. In addition, in-resonance far-field scattering patterns demonstrate intensive sidelobes in the plane of grating, explained by the spill of the mentioned Floquet harmonics. Note that $Q$-factors of GMs on finite grating are lower than on infinite ones and depend on the number of strips.

## INFINITE CIRCULAR-WIRE GRATINGS

The scattering of plane waves by free-standing infinite periodic gratings of circular cylinders or wires [see Figure 1(b)] made of metals and dielectrics has been extensively studied as a canonical scattering problem since the 1890s [26], [30], [38]–[46]. This was initiated by Hertz [47], who demonstrated that an E-polarized wave could be well reflected by a grating of metal wires while an H-polarization was passing through. His grating had approximately 70 copper wires with a radius of 0.5 cm, a period of 3 cm, and the wavelength was 66 cm. Other early experimental studies can be found in [24] and [48].

Ohtaka and Numata [40] reported, apparently for the first time, that the scattering of light by an infinite one-period grating of thin circular dielectric cylinders unusually showed narrow reflection resonances near the RA wavelengths (1). They correctly guessed that this was caused by the existence of some natural modes of the grating. However, this effect did not attract any serious attention of the community and remained unclaimed for the next 25 years.

Although the GM resonances on circular-dielectric-wire gratings in the cases of both E- and H-polarizations could be noted in some earlier papers (e.g., see [43, Fig. 2], [44, Figs. 7–10], [45, Figs. 2 and 3], and [46, Fig. 3]), they became the object of focused investigation only in 2006 in [49]–[51]. In these papers, the authors used the dipole approximation to study the extinction spectra. Experimental verification of this effect was published later in [52]. In [46], a grating of circular dielectric rods was used as an example in a fine mathematical study of the Fano-shape transmission resonances on periodic slabs. As already mentioned, the scattering resonances of various types are caused by the presence of the parent complex-valued poles of the field as a function of the wavelength. However, RAs are associated with the branch points and exist only for the infinite gratings. One of the reasons of the misinterpretation of the GM resonances in the early studies related to infinite and finite dielectric and metal wire gratings was their extreme proximity to the RA wavelengths, especially for the gratings made of thin wires with a small dielectric contrast.

Narrow resonances and high-$Q$ eigenmodes need fine computational tools that are able to provide the numerical results with many correct digits. Such accurate full-wave analysis of both wave-scattering and eignenvalue problems for the dielectric-wire gratings in free space was presented in [53] and [54] using the meshless series-expansion algorithm, whose convergence is guaranteed. It is based on the Fourier expansions in the local polar coordinates and addition theorems for cylindrical functions. A crucial point that makes it convergent (unlike [38]–[44]) is the reduction of the final matrix equation to the Fredholm second-kind type. This enabled the authors to refine earlier approximate analysis of the GM resonances in [48]–[50]. In particular, in [54], it was found that in either polarization, two GM families with field functions that are even and odd functions of $y$ with respect to the median plane of the grating exist. The former mode field is very similar to Figure 3(c) and (d), and the latter one differs by zero values at $y = 0$.

Using the same accurate algorithm, the effects of both optical LSP and GM resonances on infinite gratings of silver wires in free space (in the H-polarization case) were studied numerically in [55] and [56]. Here, the silver dielectric function was taken from [3]. Sample spectra of reflectance, transmittance, and absorbance of several different-period silver-wire gratings are shown in Figure 4 for the wires with radius $a = 48.85$ nm, i.e., in the same cross-sectional area as strips corresponding to Figure 2.

As observed in Figure 4, the LSP resonance is present as a broad Lorentz peak near 350 nm for all gratings. In contrast, the GMs usually cause Fano-shape resonances in the reflectance and transmittance and Lorentz shape in the absorbance. Similar to strip gratings (see Figure 2), if a grating has the period exactly matching the wavelength of low-$Q$ LSP resonance, then a narrow band of GM-induced transparency cuts through the much wider band of intensive reflection associated with the LSP mode (see the curves for $d = 350$ and 700 nm).

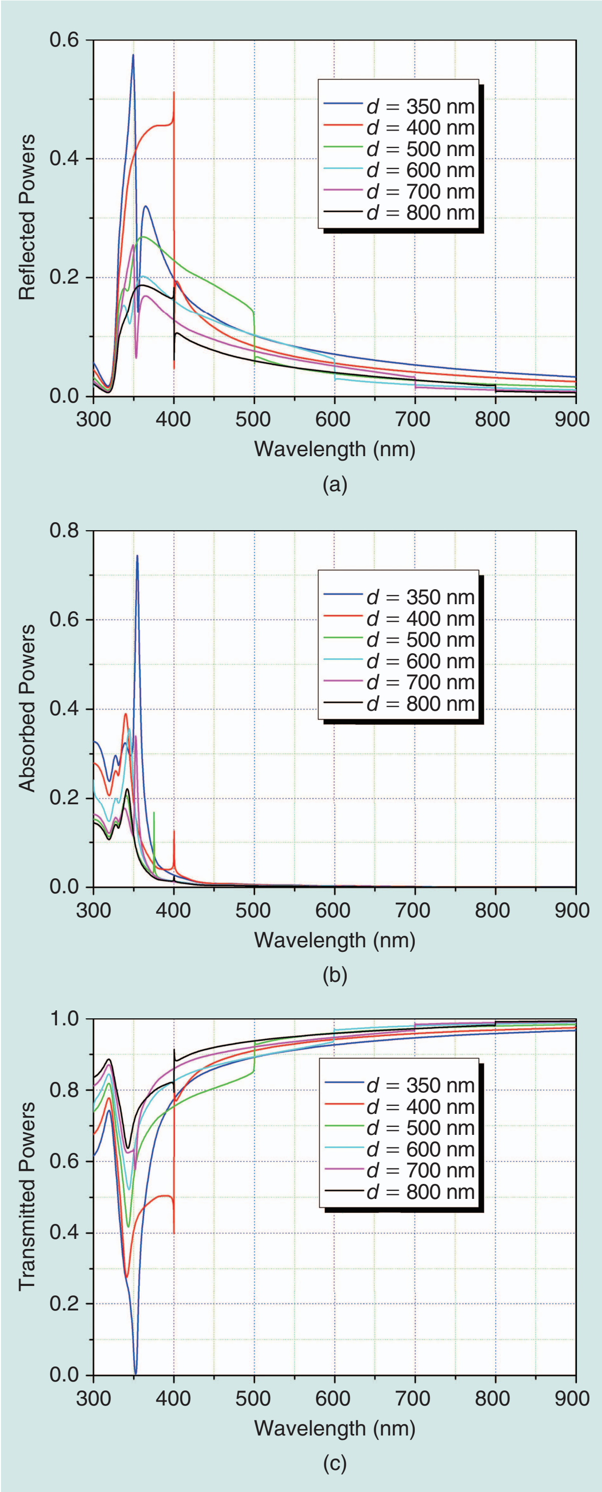


**FIGURE 4.** The wavelength dependences of (a) reflectance, (b) absorbance, and (c) transmittance powers of the different period gratings of silver circular cylinders with a radius of $a = 48.85$ nm. (Figure reused with permission from [1].)

In [54] and [55], asymptotic expressions for the complex-valued GM frequencies were derived. They showed that if the wire radius or its dielectric contrast goes to zero, then their wavelengths $\lambda^{G}_{\pm m}$ tend to purely real RA values (1) similarly to (3) for the $y$-even-mode poles and even faster for the $y$-odd-mode poles. Hence, their $Q$-factors go to infinity both for lossless and lossy wires. This has an important consequence for a grating of quantum wires made of a material with negative $\mathrm{Im}\,\varepsilon$, i.e., pumped to display material gain. The gain is able to compensate for the radiation losses and bring the mode to lasing [57]. As found in [53] and [56], the optical GMs demonstrate ultralaw thresholds of lasing that can be much lower than the thresholds of the optical LSP modes.

**Optical properties of discrete wire structures, such as corners and crosses and also in-line and two-layer finite gratings with two different periods, all made of silver nanowires, were studied.**

It is interesting to check how these optical effects manifest themselves on finite gratings that possess no RA branch points. Finite gratings of many thin wires remain a relatively unclaimed area of research, although early theoretical [38] and experimental papers [47] paid special attention to the wavelengths close to RAs. The accurate results of the numerical study obtained by the previously mentioned convergent algorithm have been published in [55] and [56] for finite silver nanowire gratings where optical LSP and GM modes exist together. The resonances on GM modes become visible in the spectra of reflectance and transmittance (see [55] for the definition of these quantities for finite gratings) provided that the number of wires is at least $N = 10$. They tend to the limit values observed for infinite gratings if $N$ gets larger.

Furthermore, optical properties of discrete wire structures, such as corners and crosses and also in-line and two-layer finite gratings with two different periods, all made of silver nanowires, were studied in [58] where the authors revealed interplay of the LSP and GM resonances. In [59], it was shown that the GM resonances can be viewed as a signature of the presence of periodicity in a cloud of random metal nanoscatterers provided that a periodic chain has 50 or more silver nanowires.

## COMPARISON BETWEEN TWO SHAPES AND TWO POLARIZATIONS IN OPTICAL RANGE

The LSP-mode optical resonances are always observed on the deep subwavelength metal scatterers with $\mathrm{Re}\,\varepsilon_{\mathrm{met}} < 0$. This is because the underlying physical phenomena essentially have electrostatic nature. Indeed, as it was shown in [5], the associated two-dimensional (2-D) static problem of a nonmagnetic cylinder in the uniform electric field possesses a set of discrete eigenvalues $\bar{\varepsilon}$ in terms of the dielectric constant. They depend on the shape of the cylinder's cross section and are negative real values. For a circular cylinder in free space, the single eigenvalue (of infinite multiplicity) is $\bar{\varepsilon} = -1$, while for a rectangle they are multiple and depend on the side lengths ratio. These eigenvalues have their projections to the H-polarized wave-scattering characteristics of the same 2-D metal scatterers whose dielectric permittivity is a function of the wavelength. As expected, the resonances on the optical LSP modes on a circular metal nanowire are found at the wavelengths near those where $\mathrm{Re}\,\varepsilon_{\mathrm{met}}(\lambda) = \bar{\varepsilon}$.

Note that in the E-polarization case, duality of magnetic and electric fields suggests that similar properties could take place if the magnetic permeability function had $\mathrm{Re}\,\mu(\lambda) < 0$. However, as all nonmagnetic objects have $\mu = 1$, there are no E-polarized LSP modes and associated with them optical scattering resonances on both metal and dielectric elements.

Keeping manufacturing issues and applications in mind, it is interesting to compare the characteristics of the gratings made of comparable silver wires and silver strips. To verify the polarization selectivity of considered gratings, it is also necessary to compare the scattering and absorption by each type of grating in two alternative polarization regimes. This comparison is presented in Figure 5 in the optical range. Here, the elementary wire and strip have the same area of cross section, and the period is fixed at $d = 800$ nm. As one can see, in the case of H-polarization [Figure 5(a)], each grating displays a broad Lorentz-shaped LSP resonance at the corresponding wavelength. Furthermore, each grating produces two supernarrow Fano-shaped GM resonances at the wavelengths slightly upshifted from the $\pm$1st and $\pm$2nd RAs in accordance to (2). Note that the LSP-mode wavelengths are considerably different; however, the GM ones agree well.

In the case of the E-polarization depicted in Figure 5(b), no resonances are visible at all. As mentioned earlier, no E-polarized optical LSP-mode poles exist for any metal grating. The GM poles, in contrast, exist for both dielectric and metal gratings in both polarizations (see [33], [40], [43], [45], [48], [49], [52], and [53]). The reason that they are not observed in Figure 5(b) can be found by examining (2): the $Q$-factors of GM poles in the E case are $|\varepsilon|^2$ times lower than in the H case that is a factor varying from 25 at 400 nm to 1,100 at 800 nm.

This invisibility of the optical GM poles on the metal nanogratings in the E-polarization scattering regime has apparently hindered correct identification of their nature because it had suggested that they might have something common to optical LSP modes, which do not exist in this regime. If a metal nanograting is placed on a dielectric substrate, the optical GM resonances become visible in the E-polarization spectra as well [32].

Thus, the best visible feature in the optical response of either type of grating in the E-polarization regime is the transmittance maxima exactly at the RAs (1). Note that the curves computed for equal-area gratings are very close to each other in the whole visible range. The largest difference in the optical spectra for two alternative polarizations is found close to the H-polarization LSP and GM resonances.

## CHAINS AND GRATINGS OF PARTICLES IN OPTICS

In 1986, Carron et al. [10] published their pioneering study into the scattering of light by large two-periodic arrays of silver nanoparticles on a substrate that produced narrow resonances near the RA wavelengths. This article nicely combined the simulations with measurements. However, at that time, this effect did not attract any reasonable attention of the research community. Ten years later, in 1996, extremely high-$Q$ complex poles near the RAs were computed in the analysis of the two-periodic gratings made of 3-D dielectric bricks (see the remarkable plot in [11, Fig. 9]). Still, these findings had not been properly understood and explained.

The next round of research into GM resonances related to one-periodic chains of spherical particles done in the group headed by Schatz. In the 2000s, they had been developing plasmonic biosensors with substrates made as two-periodic arrays of metal particles with periods usually in the range of 100–200 nm. They had been primarily concerned with painful optimization of the manufacturing and regular positioning of silver and gold particles of 10–50-nm diameters. In 2004, after several years of research, they unexpectedly noticed, in a purely theoretical analysis based on the substitution of a chain of small metal spheres with point dipoles, that if the chain period became much larger and got into the visible band, approximately 500–600 nm, then a narrow spike appeared in the simulated extinction cross section spectrum on the slope of conventional wide LSP resonance [13]. This time, the researchers were better prepared to verify their findings experimentally, and very soon they reported about catching this effect in the measurements [10]. Soon, several other groups started pursuing the topic, quickly supporting the research into GM resonances (also known as *geometric*, *lattice*, and *collective* resonances) with finer experimentation and advanced modeling [15]–[23] while looking for novel applications. All of these papers dealt with finite nanochains and nanogratings of metal spheres or short rods, and so far none of them has studied the natural modes of such chains as open resonators.

## GM APPLICATIONS IN NANOOPTICS

One of the main applications of conventional optical LSP resonances is the sensing of small changes in the refractive index of the medium, which hosts a plasmonic scatterer [8]. This is performed by measuring the position of the peak scattering or extinction and is considered as key enabling technology in the area of biological and chemical nanosensors. Therefore, it is not a surprise that remarkable properties of the recently verified GM resonances have immediately attracted the attention of scientists and engineers designing the sensors based on metallic nanogratings. In this community, such devices are (erroneously) called *RA sensors* apparently because of the nearness of the GM resonances to the RA wavelengths [23], [60]–[62]. The paper [60] is remarkable for the expressed confidence that these resonances and RAs are different phenomena (although, here, GM resonances are still interpreted as some specific plasmons). Such sensors can be based on concentric gold-ring nanograting placed on the facet of optical fiber [61]. Higher $Q$-factors make GM resonances more attractive for sensing than their LSP-mode relatives [37], [62]. As the GM-resonance wavelength is given, in the main term, just by the RA value of (1), one can expect very attractive linear dependence of the scattering peak wavelength on the refractive index. This is true provided that the analyte material is infinitely thick, while, in practice, it is usually a liquid

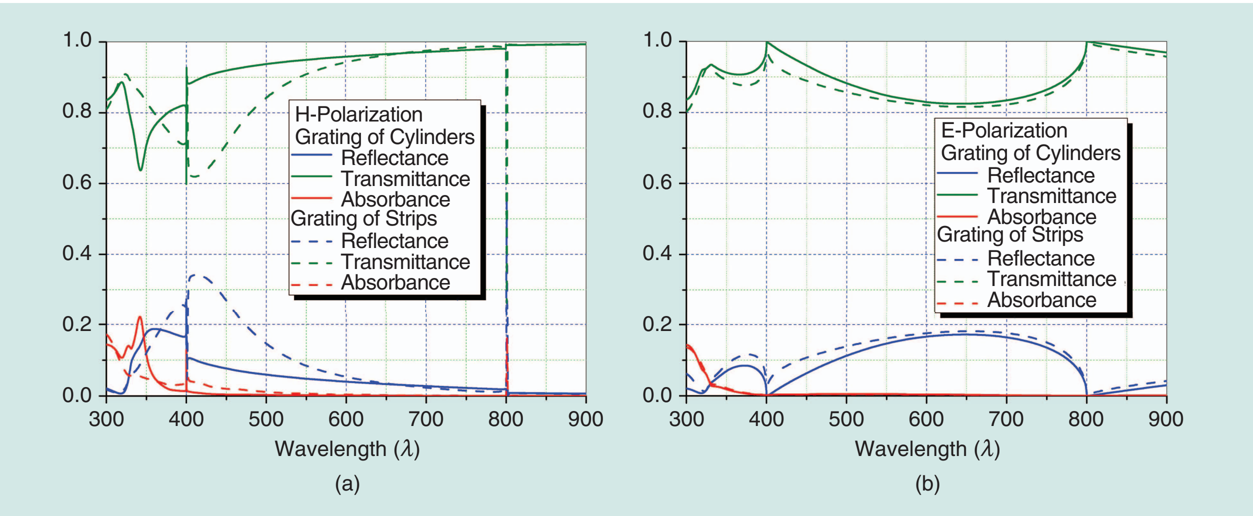


**FIGURE 5.** The reflectance, transmittance, and absorbance as a function of the wavelength for the scattering by silver strip and wire gratings with period $d = 800$ nm illuminated by the normally incident (a) H-polarized and (b) E-polarized plane waves. The strip width is $2w = 150$ nm and the thickness is $h = 50$ nm, while the wire radius is $a = 48.85$ nm.

making a finite overlay. Hence, the location of the GM peak strongly depends on the overlay thickness, so that overlays that are thinner than the wavelength are impractical. Only for thicker overlays, the refractive-index sensitivity approaches the ultimate bulk-index sensitivity value of such a sensor [23]. Other important applications are efficient nanoantennas and ultrathin absorbers for novel solar cells [63]–[68], nonlinear optics [50], and surface-enhanced Raman scattering (SERS).

## GM RESONANCES AT MICROWAVE FREQUENCIES

It is known that, at microwave frequencies, the conductivities of good metals can be considered perfect (i.e., infinite) that does not lead to gross inaccuracies. In the case of thin metal strips, it is also common to assume that they have zero thickness. As explained in the "Flat Infinite Thin-Strip Gratings" section, in such a case, the complex-valued GM poles of a flat PEC-strip grating hide in the RA branch points and no associated resonances are found in the transmittance and absorbance. However, as soon as the conductivity is assumed not to be perfect (i.e., finite) and surface-impedance conditions are used, the GM poles depart from the RAs and associated resonances become visible (see [34, Figs. 2 and 3]). Other than making the conductivity imperfect, there are many other ways to drag out these GM poles from the branch points. One of them is to tilt the zero-thickness PEC strips [69], [70], so that the grating domain obtains finite thickness. Another method is to arrange PEC strips into a comb-like configuration [71], [72] with or without a dielectric filling. The presence of GM resonances is easily detected at frequency dependences of sparse gratings' reflectance and transmittance.

Another option is to consider a flat grating made of finite-thickness PEC strips or bars [73]–[75]. Still another option is to introduce periodic perturbations along the strip edge, e.g., making it wavy [76]. Note that in some of the previously mentioned papers, GM resonances were called *trapped-mode resonances*. This term was surely borrowed from the PEC waveguide theory in connection to the resonances in wide cavities in below-cutoff waveguides.

One more option was found long ago, however, not understood properly and left without attention until recently. Indeed, in many papers published in the 1980s–1990s, the authors studied the scattering of waves by the single- and double-periodic gratings made either of PEC zero-thickness patches placed on the surface of a dielectric substrate or the periodic holes in a PEC plane supported with a substrate. Sometimes, if the frequency scans of reflection and transmission were computed with small step in frequency and using an accurate algorithm, they showed narrow resonances just below the RA values in frequency. A recent example of this effect can be seen in [77, Fig. 8].

Eventually, it was studied in detail in [78], where the existence of specific complex poles was fully acknowledged and analog of (2) can be seen in the inline equation above (3). It was also revealed in [78] that in the presence of a dielectric substrate of finite thickness, the GM frequencies shift from the free-space RAs to the values determined by the length of the principal-guided surface wave of the substrate. This type of behavior was also visualized in [79] for the GM resonances on a dielectric slab with embedded grating of circular dielectric wires.

The latter observation enables us to make an interesting prediction related to the scattering of waves of terahertz range by the gratings of graphene strips. Graphene is an electrically conducting and lossy one-atom carbon layer that can be naturally simulated as a zero-thickness resistive layer with complex-valued frequency-dependent electrical resistivity or surface impedance [80]. Suspended in the free space, a grating of graphene strips displays only the surface-plasmon resonances in the reflectance and absorbance [81], due to the inductive character of the surface impedance. This is because, similar to the PEC-strip gratings, a flat resistive-strip grating supports no magnetic surface current, and the GM poles hide in the branch points of RAs. If, however, a graphene-strip grating is placed on a thin-dielectric substrate, then powerful GM resonances must become visible in both polarizations, similar to the PEC-strip grating response studied in [31] and [75]. This prediction remains to be verified.

## CONCLUSIONS

We have demonstrated and discussed the main features of the grating or lattice resonances on the periodic arrays of circular silver wires and strips in the optical and microwave ranges. As it became clear rather recently, these resonances are caused by specific poles of the field function that should be distinguished from both conventional optical LSP poles and from the RA branch points. The associated modes have much higher $Q$-factors than those of the optical LSP modes. Therefore, their accurate analysis needs finer than usual numerical tools that are able to provide many correct digits in the unknown currents or field harmonics. Thanks to their unique properties, in the optical range, GM resonances can serve as a superior alternative to optical LSP resonances in various applications in chemical and biological sensing, photovoltaics, light harvesting, and the SERS effect. The interplay between two types of resonances depends on the angle of incidence, the grating period, and, to a lesser extent, on the cross-sectional size of each wire or strip. Choosing these parameters in an optimal manner may help design nanosensors, absorbers, and SERS substrates with substantially improved features.

We have also brought attention to the fact that the GMs and associated resonances (but not LSP ones) exist at microwave frequencies where the PEC model is valid for metal elements. These resonances have been found in a number of papers starting with [82], although their potential applications are still being studied.

Finally, we relay story that could be considered one of the most remarkable manifestations of the GM resonance effect. In the 1980s, an extremely important USSR early warning radar station somewhere in the Arctic was severely damaged by fire. Numerous capitalistic enemies could strike the defenseless country at any moment. The commander of the station was to face the military tribunal and a special government commission was examining the case. It was found that the plastic radome caught fire first. The radar was very powerful, however, it was still not able to burn the plastic. The commission came to a conclusion that the fire was caused by a metal wire mesh placed inside the radome to reinforce it. When the radar was scanning at a certain elevation angle, a mysterious resonance was excited and the wires overheated and started burning the plastic. Although the officer was found not guilty, the mystery of the resonance was not fully resolved. Now, looking at Figure 3(c), we can imagine that, in the GM resonance, a wire-mesh reinforced radome could be compared with a huge open microwave oven equipped with a megawatt source.

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