Resonance excitation of single rotons in He II by an electromagnetic wave. Spectral line shape

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The amplitude-frequency characteristic of the spectral line of electromagnetic absorption in liquid helium is measured in the frequency range 40–200 GHz at temperatures in the interval 1.4-2.75 K. It is found that in the roton frequency region a narrow resonance absorption line on a broad pedestal is observed. The results are compared with data on the roton spectrum found in neutron scattering experiments in liquid helium. The narrow line is due to the creation of a single roton. It is shown that the momentum conservation law is satisfied on account of the transfer of momentum to the superfluid component. The analogy of this effect with the Mössbauer effect is pointed out. © 2009 American Institute of Physics. [doi:10.1063/1.3266909]

I. INTRODUCTION

A recent series of experiments¹⁻³ on the interaction of electromagnetic field with liquid helium has produced a number of interesting and unexpected results that have yet to be explained in a consistent manner. One such effect is the resonance absorption and emission of microwave radiation in superfluid helium at a frequency f corresponding to the roton gap of the energy spectrum, $\varepsilon = \Delta/\hbar$. For $\Delta = 8.65$ K, which corresponds to a temperature of the order of 1.4 K, f $=\omega/2\pi \approx 180.3$ GHz. Direct evidence that the observed resonance line is due to the creation of a roton comes from the measured temperature dependence of the resonance frequency, which coincides precisely with the temperature dependence of the roton gap obtained from neutron experiments.^{4,5} Since at this frequency the photon momentum $p_{\text{pl}}=3.8 \times 10^{10} \text{ cm}^{-1}$ is many orders of magnitude smaller than the roton momentum $p_r = 1.9 \times 10^8 \text{ cm}^{-1}$, the question of how the momentum conservation law can be obeyed in such a process must be addressed.

Another point of dispute has been the very narrow width of the absorption line observed in those experiments at the roton frequency, which had not been observed previously in the neutron experiments.^{4–7} To elucidate the possible causes of this difference it is necessary to do both additional experiments and also a more careful analysis and interpretation of the experimental data. Furthermore, it has not been completely clear why absorption is observed at the roton frequency.

A study of the questions mentioned is the main content of this paper, which is devoted to further experimental study of the interaction of microwave radiation with liquid helium near the roton frequency and a more detailed comparison of the experimental data obtained with the results of neutron experiments. A possible mechanism for excitation of a single roton by an external alternating electromagnetic field is proposed on the basis of an analogy with the Müssbauer effect.⁸

II. FEATURES OF THE EXPERIMENTAL TECHNIQUE

In the present study we have used a microwave technique analogous to that described in Refs. 1-3. The difference is that, first, in order to decrease the distance between adjacent whispering gallery modes we replaced the quartz resonator with a dielectric disk resonator of leucosapphire, which has a dielectric constant several times greater than that of quartz. This made it possible to increase substantially the number of observable azimuthal whispering gallery modes and, hence, the number of experimental points in the investigated frequency range 40-200 GHz. Second, to prevent all manner of reflections of the waves, all the metallic parts (the body and frame/holder of the resonator, etc.) inside the measuring cell was coated beforehand with an absorber based on graphite powder. This made it possible to decrease the spread of values of the amplitude on the amplitude-frequency characteristic in the whole measurement range.

A resonator 20 mm in diameter was immersed in liquid helium and served as the basic measuring element working at whispering gallery modes. The exciting and receiving waveguide antennas were located in the plane of the resonator. The Q factor of the whispering gallery modes was 10^5-10^6 , which under the experimental conditions was equivalent to damping of the traveling wave by a factor of e in a liquid layer of length 6–60 km in the layer bordering the resonator. The instrumental error of the spectrometer, stabilized by whispering gallery modes, was ~100 kHz, which corre-



FIG. 1. Amplitude-frequency characteristic of the system at a temperature of 1.4 K: the resonator is found in the saturated vapor (a) or immersed in liquid helium (b). The upper dashed curve corresponds to the amplitude of the modes in the case of the vapor, while the lower dashed line is the boundary of the assumed dielectric loss in the liquid.

sponds to a temperature of $\sim 5 \times 10^{-6}$ K. The power of the radiation used in the experiments was less than 10^{-3} W, and the sensitivity of the detector sections exceeded 10^{-10} W/Hz.

III. SPECTRAL LINE SHAPE

In the experiments with the leucosapphire resonator, in a study of the passage of an electromagnetic wave of frequency 40-200 GHz through liquid helium, we also observed the features of the absorption spectrum which had been observed previously in the experiments with the quartz resonator.^{1–3} Figure 1 shows the typical spectra of the system consisting of the dielectric disk resonator, waveguide circuits, and helium for two cases: the resonator was found in the saturated vapor; the resonator is immersed in liquid helium. In both cases the temperature of the system was 1.4 K. The spectrum of this system contains a set of azimuthal whispering gallery modes separated by approximately 0.96 GHz, and the Q factor of the modes is about $10^5 - 10^6$. The spectrum also contains resonances of lower amplitude due to radial modes of the resonator and imperfections of the waveguide circuit. The radial modes can be identified both from their low amplitude and from the distance between them, which differs from the spacing of the whispering gallery modes.

After immersion in liquid helium, as can be seen from a comparison of Fig. 1a and 1b, the amplitude of the reso-

nances has decreased from a value of A_V in the vapor to A_L in the liquid on account of the increased dielectric loss (the lower dashed line in Fig. 1b). Furthermore, the resonance frequencies of the modes are shifted to lower frequencies because of the larger dielectric constant of the liquid. The evolution of one of the resonance modes (m=128) with changing temperature is shown in Fig. 2. Figure 2a pertains to the case of the resonator in the vapor, and Fig. 2b and 2c to the resonator in the liquid helium.

An important feature that can be seen in Fig. 2c is the presence of a narrow dip in the amplitude A_R on the amplitude-frequency characteristic of one of the azimuthal whispering gallery modes. This effect was observed previously in the experiments with the quartz resonator.^{1,2} Such a dip in the form a very narrow line is observed only at a certain frequency that depends on temperature and corresponds to the roton energy.¹ As is seen in Fig. 2b and 2c, it is absent from this mode at 1.6 K and present at 1.4 K; with changing temperature the resonance shifts from one mode to another.

We note that measurements made at low frequencies (40-70 GHz) show that the dielectric loss increases as the temperature approaches the λ point from both above and below, in complete agreement with the results of Refs. 9 and 10; this is due to fluctuation effects near the second-order phase transition temperature. The spectrum of superfluid he-



FIG. 2. Evolution of the resonance whispering gallery mode m = 128. The resonance is in helium vapor at T = 1.4 - 1.6 K (a); the resonance is immersed in He II at $T \approx 1.4$ K (c). The inset shows the narrow resonance absorption line on an expanded scale.



FIG. 3. Spectral line shape near the roton frequency/obtained from microwave measurements. Present study: (**D**)—quartz resonator (T=1.4 K); (**•**)—leucosapphire resonator (T=1.57 K). Data of neutron measurements⁷ (left axis): T=1.34 (\bigcirc), T=1.52 K (\triangle). The narrow resonance line is shown by the vertical line segment with amplitude A_R (see text). The horizontal segments at the 0.5 level show the widths of the roton resonance in neutron experiments⁷ and the pedestal in the present experiment.

lium at high frequencies exhibits yet another feature (Fig. 1b). The lower, straight dashed line shows the frequency dependence of the mode amplitude due to dielectric losses in the liquid. As is seen in Fig. 1b, against the background of the dielectric losses there is a smeared minimum corresponding to additional losses. This minimum broadens rapidly with increasing temperature, and its amplitude decreases, and near the λ point it becomes hard to distinguish under the conditions of the present experiment. Thus the narrow resonance absorption line is observed on a broad pedestal, and together they form the contour of the roton spectrum. One also notices that the observed minimum is asymmetric in frequency. This may be due to the circumstance that above the roton frequency the linewidth can have both phonon and roton contributions, while below the roton frequency there is only the phonon contribution.

IV. COMPARISON WITH THE RESULTS OF NEUTRON EXPERIMENTS

The experimental results presented in Figs. 1 and 2 are conveniently analyzed in terms of the relative amplitude $(A_V - A_L)/A_V$. The corresponding data for a leucosapphire resonator are shown in Fig. 3 (the filled diamonds). For comparison we also show the results of the quartz resonator measurements¹ (filled squares). It is seen from this figure that, as we have said, the spectral line shape in the roton frequency region consists of two parts: a very narrow resonance line and a broad pedestal. The amplitude of the resonance line is given in relative units $(A_V - A_R)/A_V$; the upper experimental point on the narrow line corresponds to the absorption of microwave photons.

Let us compare the results of our microwave experiments with the neutron scattering data obtained in the roton region⁷ in the same temperature interval. The roton spectrum obtained in Ref. 7 by a high-resolution technique on the IRIS neutron spectrometer is also shown in Fig. 3: at T=1.52 K



FIG. 4. Temperature dependence of the width of the narrow resonant absorption line: (\blacksquare)—quartz resonator data, (▲)—leucosapphire resonator. Below a temperature T < 1.8 K the data are limited by the resolving power of the spectrometer.

 (\triangle) and T=1.34 K (\bigcirc), close to the temperature of the present experiment. Here the neutron data have been normalized to the microwave results at the point of the maximum, where the relative amplitude was taken as unity. As can be seen in Fig. 3, there is agreement between the two experiments in the pedestal region. This indicates that the microwave scattering method, like the neutron scattering method, is sensitive to processes occurring in the gas of elementary excitations of He II.

The difference between the neutron and microwave results is that a narrow resonance line appears only in the microwave experiments. The appearance of a narrow line on the broad pedestal was unexpected, since this line had not been observed previously in the Raman scattering experiments, either.^{11,12} The apparent reason is the very high resolution of the device used in the microwave experiments. The instrumental error of the backward-wave-tube spectrometer is smaller by 4–5 orders of magnitude than that in the neutron experiments (0.1 K)⁷ and Raman experiments.^{11,12}

Figure 4 shows the temperature dependence of the linewidth of the narrow line measured with the quartz and leucosapphire resonators. It is seen that the data are in good agreement with each other and have a weak temperature dependence above T_{λ} . Below T_{λ} the linewidth of the narrow line decreases rapidly, and below 1.8 K it becomes smaller than the resolving power of the spectrometer. As to the width of the pedestal, the values obtained in the present study are in good agreement with the linewidth measured in the neutron experiments.

Figure 5 shows a comparison of the measured temperature dependence of the resonance frequency with the results that we obtained previously in microwave experiments with a quartz resonator,¹ and also with the results of the existing neutron measurements.^{4–7} It is seen that at temperatures below ~2 K good agreement is observed between both microwave measurements and practically all the neutron experiments. Near the λ point there is large scatter in the neutron data. This is apparently due to the fact that in this region the uncertainty of the energy of elementary excitations becomes comparable to their energy itself.

We also note that the disagreement with the results of neutron experiments may be due to the difference in the



FIG. 5. Temperature dependence of the roton frequency obtained from the microwave measurements (present study (\bullet), data of Ref. 1 (\blacksquare)) and from neutron measurements (data of Ref. 4 (\Box); data of Ref. 5 (\bigcirc); data of Ref. 6 (\triangle , ∇); data of Ref. 7 (\precsim)).

character of the interaction. In neutron experiments the creation of the roton is due to the neutron-roton interaction, while in the microwave experiments it is due to the interaction of the roton with the electric field. Since the probability of roton creation and the linewidth are determined by the square of the matrix element of the interaction, and the magnitude of the matrix element of the neutron-roton interaction is most likely much larger than the matrix element of the interaction of the roton with the electric field, the observation of the narrow line in a neutron experiment requires extremely high sensitivity of the measurements.

V. DISCUSSION AND INTERPRETATION OF THE RESULTS. ANALOGY WITH THE MÖSSBAUER EFFECT

Thus the microwave technique has permitted the observation of a new effect which consists in the appearance of a very narrow line at the frequency corresponding to the roton energy gap. The authors are not aware of the presence of such a narrow line in the spectra of condensed matter systems, except for the Mössbauer effect,⁸ where the spectral line is also in the form of a narrow line on a wider pedestal. As we know, the presence of the narrow peak in the Mössbauer effect is due to the fact that there is a finite probability of emission by an excited nucleus in a crystal without the transfer of energy to the phonon subsystem. Here the "rigid" correlated system that is the crystal takes on the recoil momentum, while because of its macroscopic nature it takes practically no energy away from the photon.

In our case an apparent violation of momentum conservation takes place if the roton is considered as a free particle. However, since the roton is an excitation of all the superfluid helium, it cannot be considered isolated from the superfluid part of the liquid. Because of this the momentum and energy conservation laws should be written with the transfer of energy and momentum to the superfluid subsystem taken into account:

$$\mathbf{p}_{\text{pt}} = \mathbf{p}_r + \mathbf{p}_s, \quad \varepsilon_{\text{pt}} = \Delta + \varepsilon_s \tag{1}$$

where \mathbf{p}_{pt} and ε_{pt} are the photon momentum and energy, \mathbf{p}_r and Δ are the momentum and energy of the roton, and \mathbf{p}_s and ε_s are the momentum and energy of the superfluid component. It follows from the experiments discussed above that the momentum transferred to the superfluid component is practically equal in magnitude to the roton momentum, $\mathbf{p}_s \approx \mathbf{p}_r$, while the energy transfer is very small, $\varepsilon_s \approx 0$.

We shall show that a state corresponding to these requirements can be realized in He II. If it is assumed that the absorption of a photon in the liquid gives rise to, along with the roton, a superfluid flux density $\mathbf{j}_s = \rho_s \mathbf{v}_s$ with a velocity \mathbf{v}_s (ρ_s is the density of the superfluid component), the total momentum transferred to the superfluid component is equal to

$$\mathbf{p}_s = \mathbf{j}_s V = \rho_s \mathbf{v}_s V. \tag{2}$$

where V is the volume of the system.

In such a case the total energy of the superfiuid component, with allowance for Eq. (2), is expressed as

$$\varepsilon_s = \rho v_s^2 V/2 = \rho_s^2 / (2\rho_s V). \tag{3}$$

Since the volume V is a macroscopic quantity, at a fixed momentum p_s close to the roton momentum p_r the kinetic energy ε_s of the superfluid flow that arises is extremely small. Estimates show that at V=1 cm³ and $\rho_s = \rho/2$ ≈ 0.07 g/cm³ the ratio ε_s/Δ amounts to only 10⁻²³. Thus the superfluid flow can in fact take on a finite momentum while taking away practically no energy from the roton.

Let us consider a possible microscopic mechanism for momentum transfer to the Bose. condensate through the use of a particular model. If the influence of the abovecondensate states is neglected, a single-particle Bose condensate can be described by a macroscopic condensate wave function Φ , which obeys the Gross-Pitaevkii equation¹³

$$i\hbar\frac{\partial\Phi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Phi + U_0(|\Phi|^2 - n_0)\Phi, \qquad (4)$$

where *m* is the mass of the Bose particle, n_0 is the equilibrium particle number density of the Bose condensate at rest, and U_0 is the interparticle interaction constant. The condensate wave function is normalized by the condition $|\Phi|^2 = n$, where *n* is the particle number density of the Bose condensate.

We assume that a momentum \mathbf{p} is transferred to the Bose condensate at rest. Then, according to Eq. (4), two situations are possible. First, the condensate, having acquired momentum \mathbf{p} , can be set in motion as a whole, without any excitations arising in it. In this case the stationary solution of equation (4), describing the motion of the condensate with momentum \mathbf{p} , has the form

$$\Phi(\mathbf{r}) = \Phi_a \exp(i\mathbf{q} \cdot \mathbf{r}), \tag{5}$$

where the amplitude Φ_q is related to the density of particles n_q in the moving condensate by the relation $|\Phi_q|^2 = n_q$, where **q** is an arbitrary wave vector. Then with the use of the known quantum relation for the flux density **j**, it follows from the solution (5) that $\mathbf{j} = mn_q \mathbf{v}_s$, where $\mathbf{v}_s = \hbar \mathbf{q}/m$, $n_q = n_0 - m\mathbf{v}_s^2/2U_0$. The total momentum of the condensate $\mathbf{p} = V\mathbf{j} = M\mathbf{v}_s$, where $M = mVn_q$ is the total mass of the condensate in the volume V. Thus, as was shown above in the phenomenological analysis, a finite momentum can be transferred to the condensate practically without any energy transfer.

We note that there is another possibility for the condensate to acquire a momentum \mathbf{p}_s , due to oscillations arising in it. In this case the solution of equation (4) has the form Low Temp. Phys. 35 (11), November 2009

$$\Phi = \sqrt{n_0 + \Phi_1 \exp[-i(\omega t - \mathbf{k} \cdot \mathbf{r})]}.$$
(6)

The energy of this phonon excitation $\varepsilon = \hbar \omega$ is related to the momentum $\mathbf{p} = \hbar \mathbf{k}$ by the well-known Bogolyubov relation:¹³

$$\hbar\omega = \sqrt{\frac{p^2}{2m} + \frac{hU_0p^2}{m}}.$$
(7)

Thus the Bose condensate can absorb momentum \mathbf{p} and a corresponding finite energy ε ; this does not agree with the condition of excitation of a narrow spectral line at the roton frequency. However, such excitations arising against the background of the condensate, may be responsible for the pedestal part of the spectral line shape (Fig. 3).

Thus the situation is indeed reminiscent of the Mössbauer effect, as we have said. The difference lies in the fact that in the Mössbauer effect the recoil momentum of the nucleus is transferred to the crystal as a whole. In a quantum liquid the momentum is transferred to the correlated superfluid component of the liquid.

The nature of the interaction mechanism between superfluid helium and the external electromagnetic field still requires clarification. Since the neutral helium atoms do not have an intrinsic dipole moment, it is usually assumed that the interaction with the external electric field arises on account of polarization of the helium atoms by the field. The effects due to such an interaction are quadratic in the field. It should be noted that the statement that the helium atom lacks a dipole moment pertains to an isolated helium atom. If the helium atom is found in a medium where it is surrounded by other atoms and if the distribution of atoms around it is asymmetric, then the appearance of an intrinsic dipole moment of fluctuation origin cannot be ruled out. The effects due to such a polarization are discussed in Refs. 14-17. In particular, if the roton is considered as a density fluctuation with an asymmetric distribution of atoms, then it, for the same reason, can be assigned an intrinsic dipole moment \mathbf{d}_0 . Assuming that that is in fact the case, we can write the interaction operator of a many-particle Bose system with a nonstationary external electric field $\mathbf{E}(\mathbf{r},t)$ in the form

$$\hat{V}(t) = -\int d\mathbf{r} \mathbf{d}_0 \cdot \hat{\mathbf{E}}(\mathbf{r}, t) \hat{\Psi}^+(\mathbf{r}) \hat{\Psi}(\mathbf{r}), \qquad (8)$$

where $\hat{\Psi}^{+}(\mathbf{r})$, $\hat{\Psi}(\mathbf{r})$ are the field operators for creation and annihilation of the Bose particles, and $\hat{\mathbf{E}}(\mathbf{r},t)$ is the operator of the quantized electric field of the electromagnetic wave. In Bose systems with a single-particle condensate the field operator is written in the form¹³ $\hat{\Psi}(\mathbf{r}) = \Phi(\mathbf{r}) + \hat{\varphi}(\mathbf{r})$, where $\Phi(\mathbf{r})$ is the condensate wave function, and $\hat{\varphi}(\mathbf{r})$ is the field operator describing the above-condensate excitations. When the field operators thus written are substituted into (8), the interaction operator (8) can be represented in the form of a sum of terms describing the interaction of the condensate and the above-condensate excitations with the alternating electric field. Importantly here the resulting interaction operator leads to the creation or annihilation of a single quasiparticle:



FIG. 6. Qualitative energy dependence of the density of states in He II (see text).

$$\hat{V}_{1}(t) = -\int d\mathbf{r} \mathbf{d}_{0} \cdot \hat{\mathbf{E}}(\mathbf{r}, t) [\hat{\varphi}^{+}(\mathbf{r}) \Phi(\mathbf{r}) + \hat{\varphi}(\mathbf{r}) \Phi^{+}(\mathbf{r})].$$
(9)

Expanding the electric field operator $\hat{\mathbf{E}}(\mathbf{r},t)$, the operator of the above-condensate excitations $\hat{\varphi}(\mathbf{r})$ and the condensate wave function $\Phi(\mathbf{r})$ in eigenfunctions of the momentum operator and integrating over the coordinate, we obtain an expression proportional to the delta function $\Delta(\mathbf{p}_{pt}-\mathbf{p}_r-\mathbf{p}_s)$, which ensures the conservation of momentum.

Let us explore a possible reason why the resonance is observed precisely at the roton frequency. The probability per unit time of a transition of the system from the initial state i to the final state f under the influence of an external perturbation is given by the well-known Fermi formula

$$w_{fi} = \frac{2\pi}{\hbar} |\langle f | \hat{V}_1 | i \rangle|^2 \rho(\varepsilon_f), \qquad (10)$$

where $\rho(\varepsilon_f)$ is the density of final states. The matrix element in (10) between an initial state with a photon of momentum \mathbf{p}_{pt} and a condensate at rest, and a final state with a roton of momentum \mathbf{p}_r and a moving condensate described by wave function (3) with $\mathbf{q} = (\mathbf{p}_{pt} - \mathbf{p}_0)/\hbar$, is nonzero.

Owing to the peculiar dependence of the roton energy on momentum, $\varepsilon = \Delta + (p - p_0)^2 / 2\mu$ (μ , is the effective mass of the roton), the density of states per unit volume is

$$\rho(\varepsilon) = \frac{p_r^2 \sqrt{\mu}}{\sqrt{2}\pi^2 \hbar^3 \sqrt{\varepsilon - \Delta}}.$$
(11)

The probability of excitation of a quasiparticle as a result of the absorption of a photon is proportional to the square of the matrix element and, hence, to the magnitude of the interaction of the quasiparticle with the field and to the density of final states (11). Since the density of final states of the roton tends to infinity (Fig. 6), the probability of excitation of a roton by the external field can be rather large even if the interaction of the rotation with the electric field is small. This explains why the resonance absorption is observed precisely at the frequency Δ/\hbar . With increasing frequency the density of states falls off rapidly, and the probability of creation of a roton with momentum $p \neq p_0$ is small. An analogous feature in the density of states is also present at the maximum of the energy spectrum of superfluid helium (for a maxon). Experimental investigations on this topic are being planned.

Thus to a certain approximation superfluid helium can be treated as a kind of two-level system in which the lower "level" is its ground state and the end "level" is a state with a roton of energy Δ . Under the influence of an external field of frequency $\omega = \Delta/\hbar$ one can create a large population of the

upper "level," as a result of which it becomes possible to generate radiation in the microwave range. It is also quite probable that at high pumping densities one can create conditions for observation of a kind of roton Bose-Einstein condensation in a state with finite momentum, as was predicted in Ref. 18.

CONCLUSION

We have done an experimental investigation of the influence of an alternating electric field on superfluid helium at a frequency of the electromagnetic field close to the roton energy gap. We have shown that the spectral line consists of a broad pedestal and a narrow line of resonance absorption or generation. We have established that the width of the pedestal is in good agreement with data from previous neutron experiments on the width of the roton line.

The appearance of the narrow resonance line at the roton frequency is attributed to the creation of a single roton. Since the roton momentum is several orders of magnitude greater than the momentum of a photon of the same energy, it is assumed that the momentum conservation law can be obeyed through momentum transfer to the superfluid component of He II. This mechanism is reminiscent of the Mossbauer effect, where the photon transfers momentum but not energy to the crystal as a whole. As in the Mössbauer effect, in the present case one observes a very narrow line, the like of which is not observed anywhere else in the spectra of condensed systems.

Further studies are proposed to investigate the influence of an external dc field on the structure of the spectral line shape. It is also worthwhile to investigate the inverse effect—the generation of electromagnetic radiation upon the annihilation of rotons.

The authors thank I. Adamenko, A. Andreev, V. Grigor'ev, V. Loktev, V. Maĭdanov, V. Natsik, K. Nemchenko, É. Pashitskii, S. Ryabchenko, V. Sivokon, M. Tomchenko, S. Shevechneko, G. Sheshin, and V. Yampol'skii and the participants at the International Conference on Low Temperature Physics LT25 (Amsterdam, 2008) and the Symposium on Ultralow Temperatures ULT2008 (London, 2008) for interest in this study and helpful discussions.

This study was supported by grants from CRDF (Project 2853) and UNSC (Project 3718).

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Translated by Steve Torstveit