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ELECTROMAGNETIC FIELDS AND EMISSION THRESHOLDS OF STAND-ALONE AND COUPLED TWO-DIMENSIONAL DIELECTRIC RESONATORS WITH ACTIVE REGIONS

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GENERAL DESCRIPTION

This work deals with research, using boundary-value problems for the Maxwell equations, into electromagnetic fields, frequencies and thresholds of lasing for the eigenmodes of stand-alone and coupled dielectric resonators with active regions.

Timeliness of research. Development of devices and systems that use electromagnetic waves for transmitting and processing the information heavily relies on the availability of small-size and efficient sources of short waves, from THz to the visible to UV. Today one of the key sources in these wave bands is semiconductor, crystalline and polymeric microcavity lasers. Such lasers, frequently shaped as thin disks, are equipped with active regions and pumped either with photo-pumping or with injection of carriers from metallic electrodes. In particular, such devices are considered now as the most promising sources of THz waves; they are also viewed as potential sources of single photons for the future quantum computer. Design and manufacturing of these lasers depends on complicated technologies such as dry and wet etching and molecular-beam epitaxy, and their measurements require fine spectroscopic equipment. Today there exist only several major laboratories that manufacture and measure microcavity lasers: Caltech, Yokohama National University, Laboratory of Photonics and Nanostructures of CNRS in Marcoussi, Federal Polytechnic University of Zurich, and Institute of Semiconductors of the Chinese Academy of Sciences. Therefore, it is clear that preceding modeling of such expensive devices and adequate theoretical description of the associated physical effects are critically important elements of successful research and development in this field.

However, the approaches and methods of linear modeling of microcavity lasers so far have been based exclusively on the search of complex-valued natural frequencies and associated modal fields in the *passive* dielectric resonators. Here, two approaches have been most widely used: geometrical optics (GO), known also as the billiards theory, and numerical method of finite differences in time domain (FDTD). Despite their simplicity and usefulness, each of them suffers of a number of heavy demerits. GO is not applicable to the cavities whose dimensions are comparable to the wavelength and is not able to estimate the losses and therefore the Q-factors of modes. Moreover, GO cannot grasp the discreteness of the modal spectrum of open resonator. FDTD cannot access the natural modes directly. It needs a pulsed source placed in the cavity, calculates the transient response to that source at some other point, implies the use of Fourier transform to obtain frequency dependence, and finally restores the Q-factors from the widths of resonant peaks. All this involves multiple uncontrollable errors and generally cannot guarantee the desired accuracy of modeling.

The most fundamental defect of the conventional approach is the fact that in the passive model one ignores the presence of active region. As a result, there is no chance to reproduce and quantify such a fundamental property of laser as existence of lasing threshold or explain why the light emission frequently occurs on the modes that do not possess the highest Q-factors in the absence of pumping. The attempts of building the theory able to deliver the thresholds have been linked to the quantum-mechanical nonlinear models and have not been based on the "first principles', which are the Maxwell equations with accurate boundary conditions and condition of radiation. Therefore the area and the topic of the undertaken research are timely

.

Relation to R&D programs and projects. The research related to this thesis has been done in the framework of

1. Government R&D projects of IRE NASU, "Theoretical and experimental investigation of wave processes in the devices and components of microwave and millimeter-wave bands" (code Buksir-2, #01.00U006441, 2002-2006) and "Development and application of new methods of computational radio-physics, theoretical and experimental investigation of transformations of

electromagnetic fields of the GHz and THz bands in the objects and media of anthropogenic and natural origin" (code Buksir-3, #01.06U011975, 2007-2010).

2. Program of NASU "Nanostructured systems, nanomaterials and nanotechnologies" via competitive project "Micro and nanoscale electromagnetic modeling of optical fields in resonators with active regions shaped as quantum layers, wires and dots" (code Porig, # 01.07U003983, 2007-2009).
3. Competitive project of the Ministry of Education and Science, Ukraine "Innovative numerical modeling of quasioptical focusing systems" (code Fokus, # 01.09U005351, 2009-2010)
4. Exchange program between NASU and the Royal Society, UK via joint projects «Modeling of micro and nano-scale resonators and lenses for dense photonic circuits» (#IJP-2004/R1-FS, 2004-2007) and «Advanced modeling of single and periodic active dielectric resonators for microlasers» (#IJP-2007/R1, 2007-2009) with the University of Nottingham.
5. Exchange program between NASU and TUBITAK via joint project «Innovative electromagnetic modeling of multielement quasioptical focusing systems for sub-mm and terahertz ranges» (#106E209, 2007-2009) with the Bilkent University, Ankara.
6. Exchange program between NASU and the Academy of Sciences of Czech Republic (ASCR) via joint project «Electromagnetic and numerical modeling of active and nonlinear microcavities» (2008-2010) with the Institute of Photonics and Electronics of ASCR, Prague.

It was also partially supported by the following international fellowships and scholarships:

- "Eigenvalue problems for cyclic photonic-molecule microcavity lasers," IEEE Electron Devices Society (2005),
- "Quasi-3D electromagnetic modeling of microcavity lasers and laser arrays with lowered thresholds and improved directionalities," INTAS association, EU jointly with the University of Nottingham, UK (2005-2007),
- "Advanced linear modeling of semiconductor microcavity lasers," International Visegrad Fund, EU jointly with IPE ASCR, Prague (2007-2008),
- "Electromagnetic modeling and design of dielectric lenses and resonators for the emerging photonic and THz applications," Ministry of Foreign and European Affairs, France jointly with the University of Rennes 1, Rennes (2008-2009).

Aims and tasks. The aims of research in the thesis are the building of the linear model to study the natural electromagnetic fields (modes) in the stand-alone and coupled two-dimensional (2-D) dielectric resonators with active regions, development on its basis of the numerical algorithms, computation of spectra of emission and associated thresholds for the modes in certain important types of resonators, and formulation of recommendations towards reduction of thresholds and improvement of directionality of radiation. To achieve these goals, the following problems have been considered:

- Formulation of the mathematical problem for adequate description of the natural electromagnetic fields (modes) in open resonators with active regions,
- Development of numerical algorithms for the computation of frequency spectra and thresholds of lasing, and also modal fields in the near and far zones,
- Systematic computation of the frequencies and thresholds of lasing and modal fields for the following resonator configurations:

- stand-alone circular resonators including a uniformly active one and a resonator with a partial (radially inhomogeneous) active region,
- active disk in a passive ring and an annular Bragg reflector,
- cyclic photonic molecules made of active circular disks,
- stand-alone active resonator with the spiral contour.

The object of research in the thesis is the effect of radiation of monochromatic electromagnetic waves from stand-alone and coupled dielectric resonators with active regions.

Specifically, we study the natural electromagnetic fields in two-dimensional (2-D) models of stand-alone and coupled dielectric resonators with active regions and their spectra of natural frequencies and associated material thresholds.

Methods of research used in the thesis are the following: theory of boundary-value problems of electromagnetics that views the natural modes as the solutions to the homogeneous time-harmonic Maxwell equations with rigorous boundary conditions and radiation condition at infinity. Dimensionality of these problems is reduced to 2-D using widely known approximate method of effective refractive index. For each of considered configurations, the obtained 2-D problems are equivalently reduced to homogeneous matrix equations of the Fredholm second kind. For the stand-alone and uniform and layered circular resonators and made of them photonic molecules this is achieved by using the full or partial separation of variables. For the resonator with arbitrary smooth contour the same is achieved by using the method of the Muller boundary integral equations discretized with a Nystrom-type interpolation algorithm. The eigenvalues as the roots of corresponding determinantal equations are found numerically with controlled accuracy using two-parametric iterative Newton algorithm.

Scientific novelty of obtained results is determined by the following considerations:

- The problem of natural modes of open dielectric resonators has been formulated, for the first time, in such a manner that takes into account the presence of active region and, as a result, enables one to find the modal frequencies and associated thresholds of lasing.
- For the first time an analytic connection has been found between the threshold of lasing and the mode Q-factor and the overlap coefficient between the active region and the mode electric field.
- It has been established that in a stand-alone circular disk there exist lower-order modes with high thresholds of lasing and the whispering-gallery modes with exponentially low thresholds.
- It has been shown, for the first time, that one can lower the thresholds of lasing of supermodes (coupled modes) built either on the lower-order modes or on the whispering-gallery modes by collecting the disks into a cyclic photonic molecule.
- It has been found, for the first time, that the threshold of lasing of any supermode in an active disk placed inside a passive annular reflector can be both lower and higher than in a stand-alone disk. This depends on the field overlap with the active region: the threshold greatly increases if the field is pulled into passive regions.
- It has been quantified how the deformation of the disk to a spiral resonator leads to the splitting of the modes to doublets. Here, the directionality of emission of the whispering-gallery modes improves however their thresholds grow up. The main factor is the height of the step on the contour in terms of mode wavelength.

Practical importance of obtained results. The proposed approach and the developed numerical algorithms can be used in the electromagnetic analysis of lasing modes of microresonator lasers of the UV, visible, IR, and THz ranges shaped as thin disks, cyclic photonic molecules of such disks, disks in the annular Bragg reflectors, and also thin active resonators with arbitrary contours. The established properties of the modes in such resonators significantly deepen our understanding of the thresholds of lasing. They also show the ways to the lowering of the thresholds and improving the emission directionality. Currently it is planned that some of the predicted effects will be looked for experimentally at the Ecole Normale Supeieur de Cachan, France where stand-alone and coupled polymeric microresonator lasers are studied.

Personal contribution of the candidate. All main results presented in the thesis belong to the author. Her contribution, in the co-authored papers [1-5,7-13], is in the derivation of the basic equations, development of numerical algorithms, systematic computing of the lasing frequencies and thresholds, and interpretation of numerical results; in the review paper [6] it consists of computations of sample dependences illustrating the behavior of lasing thresholds for the modes of circular resonators and photonic molecules.

Dissemination of the results. The results of this thesis have been personally presented by the author at the following scientific seminars: "Computational electromagnetics" at IRE NASU (Prof. A.A. Kirilenko), "Integral equations of electromagnetics" at the Kharkov National University of Radio Electronics (Prof. A.G. Nerukh), at the Department of Electrical and Electronics Engineering of the Bilkent University, Ankara (Prof. A. Altintas), at the George Green Institute for Electromagnetics Research of the University of Nottingham, UK (Prof. T.M. Benson), at the Photonics Research Group of the Aston University (Dr. V.K. Mezentsev), and at the Institute of Photonics and Electronics of ASCR, Prague (Prof. J. Ctyroky). They were also presented at the following national and international conferences:

- Days on Diffraction, St. Petersburg (2004, 2007)
- Physics and Engineering of Microwaves, Mm and Sub-mm Waves, Kharkov, (2004, Best Poster Paper Prize at the Young Scientist Contest)
- Mathematical Methods in Electromagnetic Theory, Dnipropetrovsk (2004)
- Antennas and Electromagnetics, Saint Malo (2005)
- Microwave and Optical Technologies, Fukuoka (2005, Invited Paper)
- Advanced Optoelectronics and Lasers, Alushta (2003,2008), Yalta (2005)
- Numerical Simulation of Optoelectronic Devices, Berlin (2005)
- Workshop on Electromagnetic Wave Scattering, Gebze (2006, Invited Paper)
- Mediterranean Microwave Symposium, Budapest (2007, Invited Paper)
- Nanosystems, Nanostructures and Nanotechnologies, Kiev (2007)
- Open Waveguide Theory and Numerical Modeling, Prague (2003), Grenoble (2005), Varese (2006), Copenhagen (2007), Eindhoven (2008)
- Transparent Optical Networks, Warsaw (2003), Wroclaw (2004), Barcelona (2005), Nottingham (2006), Rome (2007), Athens (2008), Ponta Delgada (2009)
- Waves in Science and Engineering, Mexico City (2009, Invited Paper)
- IX Young Scientists Conference on Radio Physics and Electronics, Kharkov (2009)

Publications. The results of research have been published in 46 papers including 8 papers in technical journals [1-8] and 38 papers in the proceedings and digests of international conferences, the main of which are [9-13].

Structure and size of the thesis. The thesis includes introduction, 5 chapters, conclusions, and a list of literature sources which have been used. The total thesis size amounts 189 pages, from them 20 pages are for the list of the references (177 titles).

THESIS ESSENTIALS

In introduction the timeliness of the considered topic is grounded, the aims and the tasks of the investigation are formulated, and the general characteristics of thesis are presented.

Chapter 1 is dedicated to a literature review on the thesis subject. It starts with a review of publications relevant to the topic of dissertation. First, general information on the dielectric resonators is given. Their principle of operation is based on the contrast between the refractive indices that causes the electromagnetic field reflections from the resonator boundary and thus its concentration inside the resonator. The quality factors of resonators are explained, and the areas of application are mentioned.

Further, a brief description is presented of the structure, main properties and history of investigation of microcavity lasers as open dielectric resonators with active regions. Such devices first appeared in the 1990s as miniature semiconductor sources of infra-red waves[1]; later polymeric and monocrystal lasers were proposed for the visible, ultraviolet and terahertz bands. Usually they are shaped as circular disks having diameter of 10-20 wavelengths and thickness of 0.1-0.5 wavelengths, standing on a pedestal (Fig. 1) or laying on a less optically dense substrate. Inside the disk, contained is an active region able to support the inversed population of carriers. For example, for semiconductor lasers it can be a thin quantum well, a layer of quantum dots, and also a cascade of such layers. For polymeric lasers, the whole disk can become active under pumping.

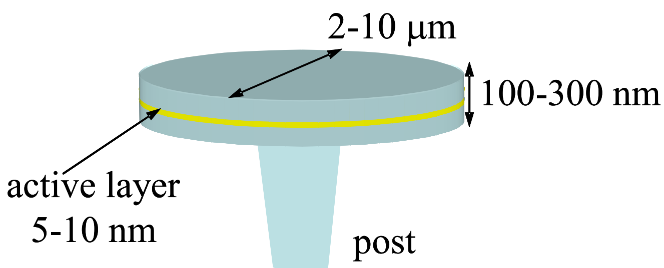

Fig. 1. Sketch of the structure of a semiconductor microdisk laser of infrared band on a pedestal.

The main properties of such lasers are the ultra-low thresholds of lasing, equidistant spectrum of lasing frequencies, and concentration of the radiation in the plane of the disk. All these properties can be explained if the working modes of the disk laser are the whispering-gallery modes, whose fields are concentrated at the inner side of the disk rim. Today trends in the research into microdisk lasers are connected to the further lowering of the lasing thresholds and the improvement of the emission directionality. To achieve these goals, researchers work on smoothing the disk rim, optimize the shape and location of the active region (pumped area), integrate active disk into an annular Bragg reflector, collect the disks into photonic molecules, and also find optimal shapes of non-circular dielectric resonators.

A review of theoretical approaches and methods used in the modeling of passive dielectric resonators is given. First of all, it is pointed out to the opportunity of the approximate lowering of the dimensionality of the original 3-D problem to two dimensions, in the median plane of a thin disk, by using the so-called method of effective refractive index. Further, discussed are the merits and demerits of two most popular methods of the optical field analysis in dielectric resonators: geometrical optics and finite-difference time-domain method.

It is noted that as the deficiencies of the mentioned methods cannot be eliminated, lately a growth of the publications using the methods of volume and boundary integral equations (IE) has been observed [6]. However, many types of the IE are not fully equivalent to the original boundary-value problem, and thus possess a set of spurious eigenvalues. If a dielectric resonator is located in free space, then the spurious eigenfrequencies of such "defective" IE models are purely real. This seriously undermines the search for the true eigenfrequencies with small imaginary parts (high Q-factors). It is emphasized that there exists IE that is free from the mentioned defects. This is the set of the Muller

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[1] S.L. McCall, A.F.J. Levi, R.E. Slusher, S.J. Pearson, R.A. Logan, "Whispering-gallery mode microdisk lasers," *Applied Physics Letters*, vol. 60, no 3, pp. 289–291, 1992.

boundary IE – a pair (in 2-D) of the IE with smooth or integrable kernels. The ways of efficient discretization of such IE are discussed.

Eventually, the opportunities and shortcomings of the model of passive dielectric resonator are discussed when applied to the investigation of lasers as open resonators with active regions. This leads to the necessity of modification of the formulation of the eigenfrequency problem for such a dielectric resonator. It is proposed to make use of the known description of the active (i.e. pumped) material as the one with negative losses [1,6]. In line with the general theorems of operator-valued function analysis, each complex-valued eigenfrequency of a dielectric resonator is analytic function of the complex refractive index[2] (see Fig. 2). Here, for a passive dielectric resonator all eigenfrequencies are located strictly on one halfplane of the complex plane. However, if the imaginary part of refractive index becomes "active", then the eigenfrequencies are allowed to migrate to the other halfplane.

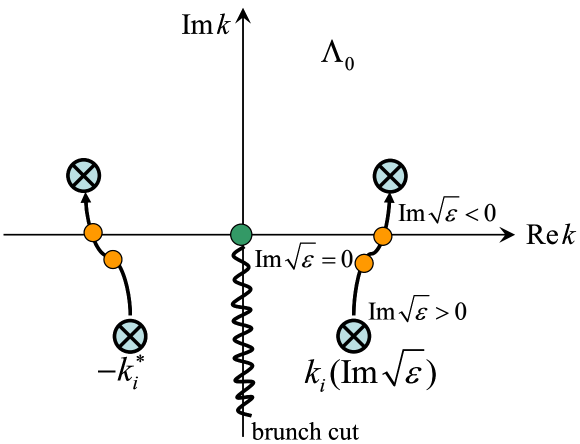

Fig.2. The trajectory of the eigenfrequency on the complex plane under the variation of the imaginary part of refractive index.

For each mode the crossing of the real axis takes place at a specific value of the imaginary part of refractive index as spatially-averaged material gain. This value corresponds to the threshold of lasing understood as emission of non-damped in time electromagnetic waves.

Therefore it is proposed to make the next step and look for the threshold value of the imaginary part of refractive index together with the real-valued emission frequency of a dielectric resonator mode as two elements of the same modified eigenvalue. Here, it is necessary to demand the continuity of the field tangential components at the boundary of the active region. As the fields of the modes having real frequencies do not grow at infinity in space, for correct formulation of the problem one may use the Sommerfeld condition of radiation. It is pointed out to a certain similarity between the proposed approach and a variant of the so-called method of generalized eigenoscillations[3], where the frequency was a known parameter and the eigenvalues were sought in terms of the complex-valued permittivity of a dielectric resonator.

Chapter 2 deals with detail consideration of 2-D models of thin circular-disk dielectric resonators, both uniformly and partially active.

First of all, 3-D problem for a thin dielectric disk is reduced to 2-D one with the aid of well-known method of effective refractive index. Here, it is assumed that homogeneous and isotropic disk of thickness d and radius a is located in free space. The real-valued bulk refractive index of the disk material is denoted as α. It is assumed that electromagnetic field depends on time harmonically as $e^{-i\omega t}$ and free-space wavenumber is $k = \omega / c = 2\pi / \lambda$, where ω is the frequency, c is the free-space light velocity, and λ is the wavelength. The effective refractive index α_{eff} is the constant of the approximate separation of variables in the disk plane and in the normal direction[4]. It is determined from the solution to the 1-D problem for the natural waves propagating on a thin dielectric layer of thickness d. As a result, in the separated 2-D problem for the field in the median plane of the disk the

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[2] S. Steinberg, "Meromorphic families of compact operators," *Archive Rational Mechanics and Analysis*, vol. 31, no 5, pp. 372-379, 1968.

[3] N.N. Voitovich, B.Z. Katsenelenbaum, A.N. Sivov, *Generalized Method of Eigenoscillations in Diffraction Theory*, Moscow: Nauka Publ., 1977 (in Russian).

[4] D. Marcuse, *Light Transmission Optics,* Computer Science and Eng. Series. Van Nostrand, New York, 1982.

bulk refractive index α is substituted with the effective index α_{eff} (Fig. 3). Therefore, α_{eff} depends on frequency, as well as type and index of the natural wave of dielectric layer if its thickness is not small.

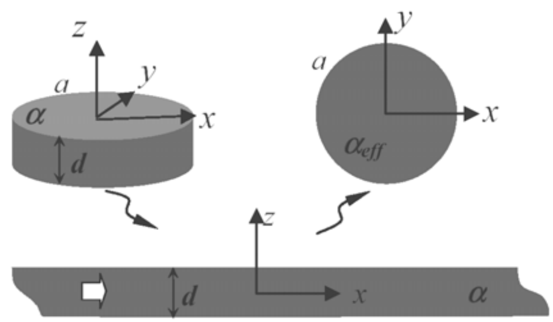


Fig. 3. Thin 3D dielectric resonator and corresponding structures of reduced dimensionalities.

The eigenvalue problem for the 2-D model of a uniformly active thin disk modified as discussed above requires the field to satisfy the 2-D Helmholtz equation with a complex refractive index $\nu = \alpha_{eff}^{H,E} - i\gamma$ inside the circle $r < a$ and $\nu = 1$ if $r > a$. At the circle boundary the field tangential component must be continuous. Besides, the fields must satisfy the condition of local energy finiteness and 2-D radiation condition of Sommerfeld at $r \to \infty$. In the analysis of active dielectric resonator we look for the modified eigenvalues, which are the pairs of positive numbers, $\kappa = ka$ and γ. The first of them is the normalized lasing frequency and the second is the threshold value of material gain.

Implementation of the method of separation of variables enables one to establish that all the modes in a circular dielectric resonator split into independent orthogonal families with respect to the azimuth index m = 0,1,2,… and are twice degenerate if $m > 0$. For each family modes, transcendental equations are derived whose roots generate discrete values of κ and γ [1]. Asymptotic analysis of these equations shows that the lower modes whose family index $m \ll \kappa_{mn}^{H,E}$ have large radiation losses, and their thresholds are

$$\gamma_{mn}^{H,E} \approx \ln[(\alpha+1)/(\alpha-1)](\pi / 2\kappa_{mn}^{H,E}) \quad (1)$$

If the opposite is true, and $m \gg \kappa_{mn}^{H,E} \gg m/\alpha$, then the corresponding modes are the whispering gallery modes whose thresholds decrease exponentially with frequency or index m,

$$\gamma_{mn}^{H,E} \approx const\ e^{-2m\ln(2m/\kappa_{mn}^{H,E})} \quad (2)$$

Here, the asymptotic expression for the normalized frequency of lasing is as follows (n being the radial mode index):

$$\kappa_{mn}^{H,E} \approx (\pi / 2\alpha)(m + 2n \mp 1/2)\,. \quad (3)$$

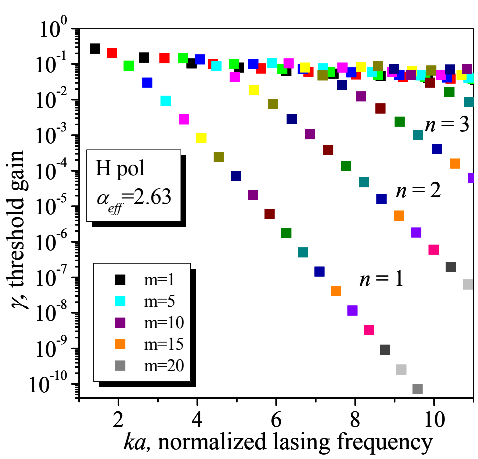


Fig. 4. Normalized frequencies and thresholds of lasing for the H_z-polarized modes in a uniformly active thin disk.

In Fig. 4 presented are the results of computation of lasing frequencies and thresholds $(\kappa_{mn}^{H}, \gamma_{mn}^{H})$ for the disk of GaAs/InAs having thickness of 200 nm, without account of the dispersion of α_{eff}, which is taken as the propagation constant of the principal wave of the dielectric layer having the same thickness and assuming that $\lambda = 1550$ nm. In this case, the effective refractive index is $\alpha_{eff} = 2.63$ for the H_z-polarized modes. One can clearly see the hyperbola $\gamma \approx const/\kappa$ corresponding to (1), saturated with lower modes of all families whose thresholds $\gamma > 0.01$. Below this hyperbola the eigenvalues form layers corresponding to radial index n in accordance to (2). These are whispering gallery modes whose fields experience almost total internal reflection from the disk rim. Those of them whose fields have single variation in radius (n = 1) form “elite” of the lasing modes: they possess the lowest thresholds in agreement with (2). It can be noted that accurate account of the α_{eff} dispersion (see [2]) changes the obtained numerical results only quantitatively and mainly in the lower part of the frequency range.

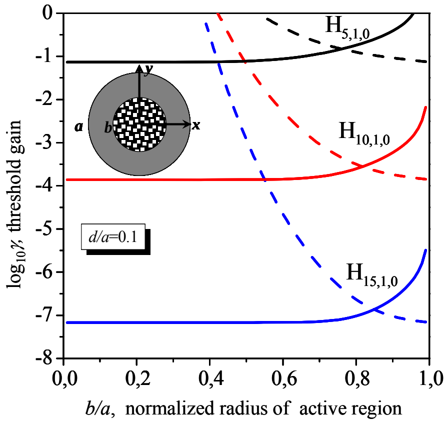


Fig. 5. Lasing thresholds for the whispering gallery modes of the family $H_{m,1}$ in the disk with radially non-uniform gain. Solid lines are for active ring and dashed lines are for the active circle in the center of the disk, $\alpha = 3.374$ and $d/a = 0.1$.

Further, considered is the eigenvalue problem for a 2-D circular dielectric resonator having active region shaped as either a circle of smaller radius in the resonator center or a ring adjacent to the resonator rim (Fig. 5). The prototypes of such dielectric resonators are the microdisk lasers of injection type with the electrodes located at the disk center or along its rim, respectively. In these cases the density of injected carriers, and hence the material gain, has maximum value below the electrode and rapidly decreases off this region.

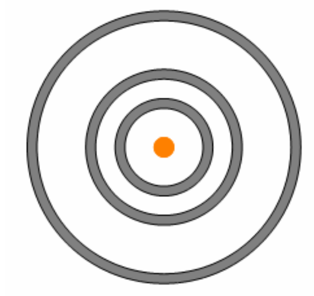
For simplicity, we assume that material gain γ either is a constant inside the circle of radius $b < a$ and zero off this circle or wise versa. Then the refractive index inside the disk is taken as $\nu = \alpha_{eff}$ in the passive region and $\nu = \alpha_{eff} - i\gamma$ in the active region. Modal characteristic equations are derived using the separation of variables [2]. Numerical study shows that the lasing frequencies of the whispering gallery modes are very close to their values in the uniformly active disk, but the thresholds vary greatly. The dependences of thresholds for the $H_{m,1}$ family modes on the normalized radius of active or passive inner circular region are shown in Fig. 5.

One can see that reducing the radius of a centered active region leads to the catastrophic growth of lasing thresholds. If, contrary, the active region is shaped as an outer ring then the thresholds start growing only if the ring becomes narrower then the domain occupied with intensive field spots for the given mode. As these spots are stronger concentrated at the disk rim for the modes having larger azimuth indices m, the active zone for these modes can be done remarkably narrow without the effect on the lasing thresholds.

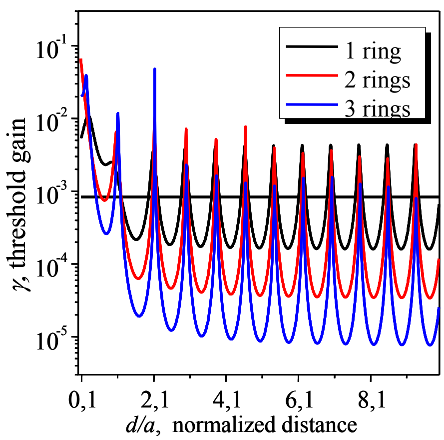
Chapter 3 considers 2-D models of multilayered open dielectric resonators having circular symmetry and containing partial active region (Fig. 6). As the modes of different partial regions become optically coupled in such configuration, they are usually called supermodes.

At first, the properties have been studied of the dipole-type supermodes in the simplest configuration of this sort: a uniformly active disk inside one passive ring with the gap between them filled with air. As the radiation of real microdisk laser is concentrated in the disk plane, the introduction of a ring enables one to expect a reduction of radiation losses. Under pumping, this may lead to the reduction of lasing threshold that is especially important for the lower order modes.

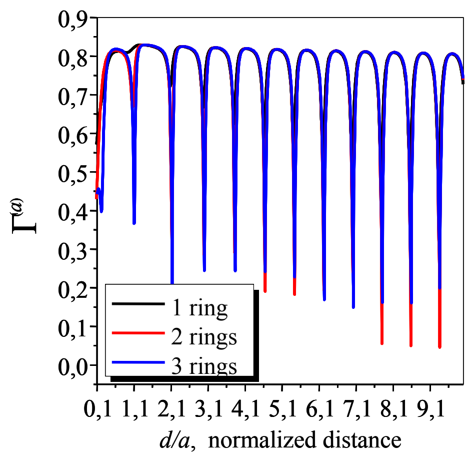
Due to the circular symmetry, the separation of variables brings characteristic equations whose roots can be found numerically [7]. The computations have concerned the dependences of lasing frequencies and thresholds for the dipole supermodes on the widths of the air gap and the ring; the modal fields have been visualized as well. It has been found that the thresholds can be both lower and higher than in a stand-alone active disk. Here, high values of threshold always correspond to the pulling of the modal field into the passive ring or the air gap.



(a)



(b)



(c)

Fig. 6. Active disk in the resonator with a passive ABR (a) and thresholds of lasing (b), and overlap coefficients (c) between the active region and electric field for the supermode $H_{7,1,p,(q)}$ as a function of the distance between the disk and the first ring.

Further, we have studied the supermodes of the whispering gallery type in an active disk placed in the center of a passive annular Bragg reflector (ABR). It has been shown that the modes whose frequencies are in the stopband of ABR obtain lower thresholds. Adding one period of ABR, i.e. a pair of rings with different refractive indices, is able to lower the threshold by an order of magnitude if the contrast between the refractive indices is large enough. However, one can also observe the situations where the threshold of lasing sharply increases and becomes much larger then the corresponding value in a stand-alone disk.

For the explanation of observed effects, we have considered the Poynting Theorem (a.k.a. Optical Theorem) for the non-attenuating in time natural modes of lasers as open dielectric resonators and resonators with uniform or partial active regions [7]. A simple and rigorous analytical connection has been found between the threshold of lasing of the *j*-th mode, from the one side, and its quality-factor and overlap coefficient of the electric field of this mode with the active region, from the other side,

$$\gamma_j = \alpha\left[\Gamma_j^{(a)}(\gamma_j)Q_j^{(a)}(\gamma_j)\right]^{-1}. \qquad (4)$$

Here, the quality-factor $Q_j^{(a)}$ of the mode in the active open resonator is understood as the ratio of the energy stored in it to the energy lost for radiation, overlap coefficient $\Gamma_j^{(a)} \le 1$ is the ratio of electric field energy in the active region to the total energy in the resonator volume, and the volume of the open resonator is the inside region of the minimum sphere that contains all resonator's elements. If neglecting the values of the order $O(\gamma_j^2)$ in the right-hand part of (4), then one may substitute the same mode field in the corresponding passive resonator, i.e. neglect the presence of active region. Thus, to have low threshold of lasing under pumping it is not enough to have high quality-factor of the mode without pumping. It is equally necessary to provide a good overlap between the active region and the mode electric field.

Numerical results presented show that if the frequency and threshold of lasing are found from the rigorous characteristic equation, then the relation (4) is satisfied with machine precision. For the low-threshold modes such as whispering gallery modes of an active disk in a passive ABR, the dependences of modal thresholds and overlap coefficients on the geometrical parameters (Fig. 6) demonstrate mutually inverse behavior that is in agreement with (4).

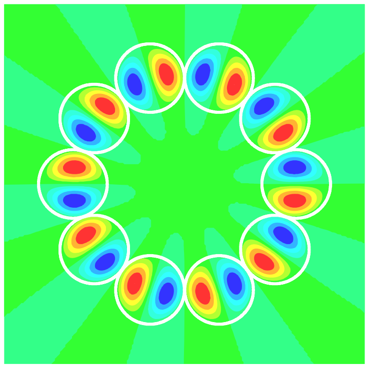


Fig.7. Near field of the dipole type supermode $H_{1,1}^{(M)}$ of the maximally anti-symmetric class in photonic molecule of $M = 10$ active disks.

In **Chapter 4**, studied are the 2-D models of the coupled active circular resonators shaped as cyclic photonic molecules (Fig.7). As the simplest structure of this type, the molecule of two identical active circular resonators is considered. This is a 2-D model of the pair of thin disks located in the same plane. Such geometry has two lines of symmetry and therefore its supermodes split into four orthogonal classes with different symmetry properties relatively to these lines [3]. For each class supermodes, the use of partial separation of variables, together with boundary conditions and conditions of local power finiteness and radiation, leads to homogeneous infinite-matrix equations of the Fredholm second kind. Thanks to the spectral equivalency with original problem, the eigenvalues coincide with the zeroes of corresponding determinant. They can be found numerically after truncating the matrix to finite order. The convergence of approximate eigenvalues to the accurate ones is guaranteed by Fredholm nature of the matrix operator.

The computation of the lasing eigenvalues for the supermodes of all four classes build on the whispering gallery modes in each disk have shown that the thresholds can be both higher and lower then the threshold of the same mode in stand-alone disk depending on the distance between the disks.

These studies have been extended to the H_z-polarized modes of more complicated coupled dielectric resonators shaped as cyclic photonic molecules of M active identical disks. In this configuration, the number of supermode classes having different symmetry equals to M+1 or M+2 depending on the parity of M. Here, the most interesting are the supermodes that possess maximum degree of symmetry or anti-symmetry. For each symmetry class the lasing eigenvalue problem has been reduced to homogeneous infinite-matrix equation of the Fredholm second kind. Numerical investigation of the frequencies and thresholds of lasing has demonstrated that the threshold can be significantly lowered by tuning the distance between elementary resonators.

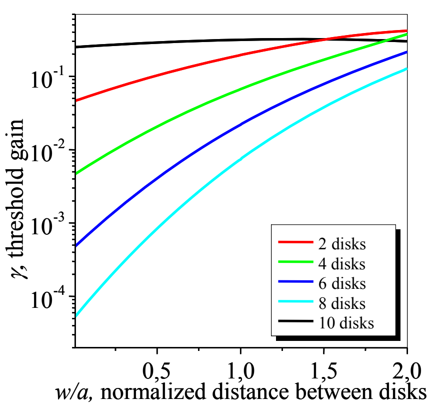
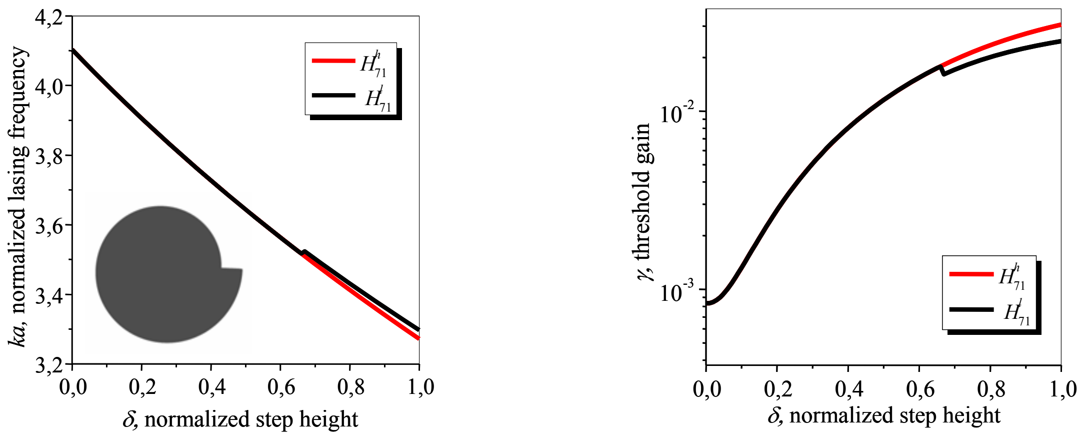


Fig. 8. Dependences of the lasing thresholds of the dipole supermodes $H_{1,1}^{(M)}$ of the maximally anti-symmetric class on the normalized rim-to-rim distance.

Here, we have found a considerable difference between the properties of supermodes built on the lower (monopole and dipole) modes [5] and the whispering gallery modes [4]. In the first case the lowering of the threshold by collecting small disks in a cyclic photonic molecule takes place only for the supermodes of the maximally anti-symmetric class (so called $\pi-$type supermodes). This effect has non-resonant nature and is stronger for the smaller rim-to-rim distance. Besides, adding new pair of disks to photonic molecule lowers the threshold of such supermodes approximately by an order of magnitude. This is explained by a more complete canceling of partial fields radiated by adjacent disks in the anti-phase to each other (Fig. 8).

In the second case, the threshold of a whispering gallery mode of any symmetry class is low from the beginning because elementary disks are quite large. It has been found that the threshold can be lowered further, if one tunes the rim-to-rim distance properly. This effect is observed if that distance is comparable to the disk radius. It has resonant character: the accuracy of tuning should be of the order of 0.1 of disk radius. This is explained by a complicated interference of the partial fields radiated by the disks of optically large dimensions.

Chapter 5 deals with 2-D Muller boundary integral equations in the analysis of electromagnetic field in a homogeneous dielectric resonator having arbitrary smooth contour[5] and their discretization with efficient numerical algorithm [8]. This algorithm is further applied to the analysis of lasing frequencies and thresholds and the fields of natural H_z-polarized modes in the thin uniformly active dielectric resonator having spiral contour (Fig. 9).



(a) (b)

Fig. 9. Dependences of the normalized frequencies (a) and thresholds of lasing (b) on the normalized height of step in a spiral resonator for the modes of the doublet $H^{h,l}_{7,1}$. The step angular width is $\beta = \pi/100$, the refractive index is $\alpha_i = 2.63$, and the number of nods of the quadrature scheme is $2N = 800$.

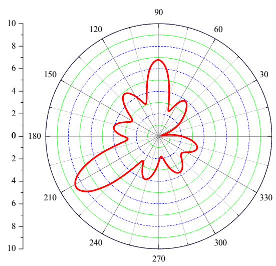
In each polarization, the eigenvalue problem for the Maxwell equations with additional conditions can be equivalently reduced to two coupled boundary integral equations of Muller using the Green's formula. Discretization of these integral equations is done by the method of qudratures (a.k.a. Nystrom method)[6]. This method is based on the approximation of integrals with finite sums using the corresponding quadrature formulas, which take into account the properties of the integrand functions including their singularities. Some of the kernels of obtained equations have logarithmic singularities that should be separated. Further, a quadrature formula with equidistant nodes is applied to the numerical integration of the logarithmic parts of involved integrals. Here, the integrand function is approximated by a trigonometric polynomial. The remaining smooth parts are integrated using the trapezoidal rule.

To parameterize the contour of the spiral resonator, a piece-given function of the polar angle proposed in the work[7] has been used. This function has two parameters: δ is the spiral step height normalized by the minimum resonator radius and β is the step inclination (equivalently, 2β is the step angular width). This is a smooth function, i.e. continuous one with a continuous derivative; however its second derivative (contour's curvature) has finite jumps at two points. Therefore, the rate of

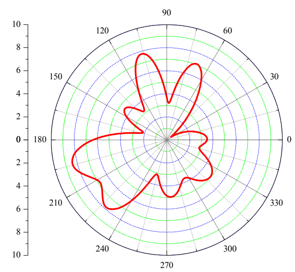
[5] C. Muller, *Foundations of the Mathematical Theory of Electromagnetic Waves*, Berlin: Springer, 1969.
[6] D. Colton, R. Kress, *Inverse Acoustic and Electromagnetic Scattering Theory*, Berlin: Springer, 1998.
[7] D. Kouznetsov, J. Moloney, "Efficiency of pump absorption in double-clad fiber amplifiers. Broken circular symmetry," *J. Optical Society of America B*, vol.19, pp. 1259-1263, 2002.

convergence when increasing the order of interpolation is not high. If one uses a standard desktop computer, obtaining 3-4 correct digits in the eigenvalue needs around one or two hours of computation.

The computations have shown that the deformation of the circular contour into the spiral one results in the splitting of originally twice degenerate modes with the azimuth index $m > 0$ to doublets having different eigenfields (Fig.10).



(a)



(b)

Fig. 10. Far-field patterns $|H_z|$ of the whispering gallery modes in a spiral resonator: $H_{7,1}^{l}$ (a) ka = 3.2962, γ = 2. 52*10^{-2}; $H_{7,1}^{h}$ (b) ka = 3.2714, γ = 3.048*10^{-2}, N = 400, spiral step height δ= 1, $\beta=\pi/100$, α_i=2.63.

We denote the modes in a doublet as $H_{m,n}^{h}$ and $H_{m,n}^{l}$, where two lower indices correspond to the numbers of field variations in azimuth and in radius similarly to the original circular resonator, and the upper index (*h*igh, *l*ow) corresponds to the value of the lasing threshold. If the spiral step height is decreased, then the modes of the doublet get closer both in terms of the wavelength and the threshold (they coincide if δ= 0). In Fig. 9 presented are the step-size dependences of the lasing frequencies and thresholds for the whispering gallery modes of the doublet $H_{7,1}^{h}$ and $H_{7,1}^{l}$. One can see that the step of the height $\delta = 0.5$ leads to the growth of the threshold by an order of magnitude for the both modes of the doublet, with respect to the circular resonator. In the far zone, both modes fields of this doublet have one or two well-shaped main beams (Fig. 10) in contrast to 2*m* identical beams for the mode having azimuth index *m* in the circular resonator.

CONCLUSIONS

In the thesis, new approach has been developed to the study of an important problem of electromagnetics. This means the development of a linear electromagfnetic model able to characterize not only the frequencies but also the thresholds of lasing for the natural electromagnetic fields in dielectric resonators with active regions. Within this approach, modified eigenvalue problems for several important types of two-dimensional dielectric open resonators have been considered. Among them, there are stand-alone uniformly and non-uniformly active circular resonators, coupled active circular resonators, and active non-circular resonators

The study of the considered problems has been based on the following elements:

- Introduction of the active region containing an active material characterized with “negative absorption” and continuity conditions for the tangential field components at the active region boundary,

- Formulation of the mathematically correct problem for the eigenvalues consisting of the ordered pairs of real-valued numbers: modal frequencies and thresholds of lasing (imaginary parts of the refractive index of the active-region material),
- Application of the widely known method of effective refractive index to the approximate reduction of the problem dimensionality for thin dielectric resonators,
- Reduction of the eigenvalue problems to the transcendental equations or determinantal equations for the Fredholm second kind matrices that guarantees the discreteness of the eigenvalues and convergence of the numerical search methods,
- Use of the Muller boundary integral equations in the eigenvalue problem for the resonator with arbitrary smooth contour and exponentially convergent Nystrom-type method for their discretization,
- Implementation of two-parametric Newton method for iterative search of eigenvalues as the roots of the obtained transcendental or determinantal equations,
- Numerical control of the fulfillment of the power conservation law (Poynting Theorem) for the natural lasing modes of active dielectric resonators,
- Systematic verification of the fulfillment of the boundary conditions for the modal fields, as well as their behavior when studied resonators transform to simple configurations.

The main results of the research are as follows:

1. The formulation of the eigenvalue problem for the natural modes of open dielectric resonators has been modified, for the first time, in such a manner that it takes into account the presence of the active region and, as a result, enables one to find the frequency spectra and material thresholds as elements of eigenvalues.
2. Based on the Maxwell equations, for the first time a simple and rigorous analytical connection has been established between the lasing threshold of an open resonator natural mode and its quality factor and overlap coefficient between the active region and the mode electric field.
3. Efficient numerical algorithms have been developed for the computation of the lasing frequencies and thresholds and the modal fields in the near and far zones, for the uniformly and partially active circular resonators, cyclic photonic molecules of such resonators, and two-dimensional active resonators with arbitrary smooth contours.
4. It has been established that a thin stand-alone microdisk supports lower modes having high emission thresholds and whispering-gallery modes having exponentially low thresholds.
5. It has been demonstrated that one can lower the thresholds for the supermodes (coupled modes) built both on the lower modes and the whispering-gallery modes by collecting microdisks into cyclic photonic molecules.
6. It has been found that the threshold of lasing in a microdisk placed inside a passive annular reflector can be both lower and higher than in a stand-alone disk. This depends on the overlap between the modal electric field and active region. The threshold grows up if the field is pulled into a passive region.
7. It has been shown that the deformation of a thin active microdisk into a spiral resonator leads to the splitting of the lasing modes to doublets. This splitting has been accurately quantified in terms of both frequency and threshold. Here, the directionalities of emission of the whispering-gallery modes increase however their thresholds drop. The main factor is the height of the step on the resonator contour in terms of the mode wavelength.

The results obtained enable one to consider the shape and location of the active region as engineering parameters, which can be used to manipulate with the lasing thresholds of the modes in dielectric resonators. Besides, they show the ways for the lowering of the threshold and the improvement of the directionality by changing the shape of the cavity contour (for stand-alone resonators) and by using the symmetry (for coupled resonators). Therefore they can be applied for the interpretation of the experimental data and for the design of the promising configurations using preliminary computer-aided simulation of microlasers.

Main publications related to the thesis

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