# Ultralow Lasing Thresholds of $\pi$-Type Supermodes in Cyclic Photonic Molecules Composed of Submicron Disks With Monopole and Dipole Modes

Elena I. Smotrova, *Student Member, IEEE*, Alexander I. Nosich, *Fellow, IEEE*, Trevor M. Benson, *Senior Member, IEEE*, and Phillip Sewell, *Senior Member, IEEE*

***Abstract*—Cyclic photonic molecules (CPMs) composed of an even number of identical submicron-size circular-disk active resonators in free space are studied in two dimensions. Cold-cavity lasing frequencies and threshold values of material gain are extracted from an electromagnetic eigenvalue problem with exact boundary and radiation conditions. We show that assembling several small cavities in a tightly packed CPM efficiently lowers the thresholds of optically coupled nonwhispering-gallery modes of the lowest values (i.e., 0 and 1) of the azimuth index in each cavity provided that high antisymmetry is arranged.**

***Index Terms*—Dipole mode, lasing eigenvalue problem, linear threshold, microcavity laser, monopole mode, photonic molecule.**

## I. Introduction

FIRST reported in the 1990s, semiconductor microdisk lasers on etched pedestals or low-index substrates and equipped with quantum wells or quantum dots are intensively investigated today. This is because of their amazing properties and expectations that these miniature sources of light will be in great demand in future high-density photonic circuits. Periodic frequencies, very low thresholds, and the predominantly in-plane light emission of microdisks are explained by the whispering-gallery (WG) character of the lasing modes. Recently, photonic-molecule (PM) lasers have been demonstrated, i.e., arrays of microrings and microdisks with coupled WG modes split into close multiplets of "supermodes" [1], [2]. Here, cyclic PMs stand out of all possible configurations due to a higher geometrical symmetry. A theoretical analysis of cyclic photonic molecules (CPMs) has led to prediction of higher $Q$-factors (pump OFF) [3] and lower lasing thresholds (pump ON) of the WG-type supermodes [4]. For each WG supermode, this effect needs a fine tuning of the distance between the cavities, to a certain fixed value that is larger than the disk radius and comparable to the wavelength.

In contrast to [1]–[4], here we numerically study a CPM made of active disks each so small that it can support only the lowest order modes, $H_{01}$ and $H_{11}$. Note that these two modes never display WG-mode features connected to the almost total internal reflections. Therefore, their linear thresholds for a stand-alone disk are much higher than those for the WG modes. Nevertheless, these lowest modes admit a simple and efficient engineering: by assembling those in a circle and bringing them close to each other, one can lower the lasing thresholds almost to the same values as for the WG modes.

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E. I. Smotrova and A. I. Nosich are with the Institute of Radio-Physics and Electronics NASU, Kharkov 61085, Ukraine (e-mail: elena_smotrova@yahoo.com).

T. M. Benson and P. Sewell are with the George Green Institute for Electromagnetics Research, University of Nottingham, Nottingham NG7 2RD, U.K.

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## II. Formulation

We consider a CPM of $M$ identical microdisks. We suppose that each disk has thickness $d$, radius $a$, and real-valued pump-OFF refractive index $\alpha$. Separation between adjacent disks is denoted as $w$, and their centers are located at the vertices of a regular polygon. Time dependence is implied as $e^{-i\omega t}$, and the free-space wavenumber is $k = \omega/c = 2\pi/\lambda$, where $c$ is the velocity of light and $\lambda$ is the wavelength. Note that macroscopic description with refractive index is valid unless the size of material sample is smaller than collision-free range of an electron's run path, i.e., around 10 nm. Reduction of electromagnetic field equations to the two-dimensional (2-D) model in the PM plane is performed by using the effective refractive-index approximation, which implies that the disks are thinner than the radius and the wavelength, $d \ll a, \lambda$.

For an accurate quantification of the lasing modes in a flat CPM of optically coupled active (pump ON) microdisks, we use the lasing eigenvalue problem (LEP) introduced in [5]. LEP is specifically tailored to extract not only the lasing frequencies but also the cold-cavity (i.e., linear) thresholds from the field equations. We have already applied it to the WG modes in disk lasers [6], twin-disk lasers [7], and two-disk PM lasers with one active disk and one passive disk [8].

In 2-D, two different polarizations can be treated separately, with the aid of either the $E_z$ or $H_z$ field components, and the $H$-polarized case is of primary importance because of a larger value of effective index $\alpha_{\text{eff}}$. The field function must satisfy the 2-D Helmholtz equation where, inside disks the bulk refractive index $\alpha$ is replaced with the complex-valued parameter $v = \alpha_{\text{eff}} - i\gamma$ (here, we assume that material gain $\gamma > 0$ is uniform across the disks). At the disk rims, the continuity conditions hold for the tangential field components. The Sommerfeld radiation condition is imposed at infinity. In the LEP, we look for the pairs of real numbers—normalized frequency $ka$ and dimensionless material threshold gain $\gamma$, which generate nonzero electromagnetic fields. Note that the gain per unit length (the traditional quantity used in the study of Fabry–Pérot laser cavities, measured in inverse centimeters) can be obtained as $g = k\gamma$.

A CPM of $M$ identical microdisks has $M$-fold symmetry. Therefore, all possible supermodes of such a PM split into several independent symmetry classes [9]. Note that if $M$ is an even number, there exists a specific class of nondegenerate modes, whose field functions are the same in adjacent resonators, except for being in antiphase. This justifies calling such supermodes "$\pi$-type" ones, similarly to the famous modes in the WWII microwave cavity magnetrons. Further, another set of symmetry lines going through the centers of resonators splits the $\pi$-class to two orthogonal ones, $\pi^+$ and $\pi^-$, according to the modal field parity inside an individual cavity. Supermodes of these classes may have reduced leakage to free space due to cancellation of radiative contributions from individual disks.

## III. Method of Numerical Solution

We expand the field function inside each cavity in the Fourier series in terms of the angular functions in the local polar coordinates. After applying the boundary conditions, using the addition theorem for cylindrical functions, and satisfying the requirements of symmetry or antisymmetry we obtain an infinite-matrix equation. For each mode symmetry class, it can be written as a Fredholm second kind equation $[I+G(\kappa,\gamma)]X = 0$, where $X = \{x_m\}_{m=(0)1}^{\infty}$ is the vector of field expansion coefficients, $I$ is identity operator, and $G = \{G_{mp}\}_{m,p=(0)1}^{\infty}$ is a compact operator. The LEP eigenvalues can be found as roots of truncated equation, $\mathrm{Det}^N[I+G(\kappa,\gamma)] = 0$. Convergence of these roots to the exact eigenvalues of infinite matrices is guaranteed when the truncation order $N$ is increased [7].

## IV. Results and Discussion

In computations, we assume that the semiconductor system is GaAs–InP and $d = 100$ nm. Therefore, at first we take $\lambda = 1.55$ $\mu$m and find the bulk refractive index as $\alpha = 3.374$ and the $H$-polarization effective index as $\alpha_{\mathrm{eff}}^{H} = 1.967$. Then we compute the actual lasing frequencies and thresholds to an accuracy of $10^{-5}$ by using a two-parameter secant-type iterative method [6]. Here, one may keep refractive indexes constant; however, their dispersion can be accounted for with the aid of adaptive algorithm described in [6].

The "monopole" mode $H_{01}$ and two orthogonal "dipole" modes $H_{11}^{\pm}$ exist in the smallest stand-alone cavities, which have submicron size. In our case, $(ka)_{H_{01}} \approx 1.206$ and $(ka)_{H_{11}} \approx 1.914$, hence $a_{H_{01}} \approx 297$ nm and $a_{H_{11}} \approx 472$ nm, that is actually at the boundary of the thin-disk model validity range. True WG modes exist in larger disks, with much larger values of the normalized wavelength $m > ka > m/\alpha \gg 1$, and display exponentially low thresholds $\gamma \sim \mathrm{const}\, e^{-ka}$. In contrast, non-WG modes in a stand-alone disk have much higher thresholds $\gamma \sim \mathrm{const}\,(ka)^{-1}$ [6]; for the lowest modes they are record-high: $\gamma_{H_{01}} = 0.3921$ and $\gamma_{H_{11}} = 0.2959$.

In Figs. 1 and 2, we show the dependences of the lasing thresholds and frequencies on the relative rim-to-rim distance $w/a$ for the supermodes $H_{01}$ of the $\pi^+$-type mode class (maximally antisymmetric ones) in CPMs of two to ten microdisks. For compactness, we denote this supermode as $\pi^+(H_{0,1})_M$. The curves show that, if the separation gets smaller, supermodes of this type are progressively blueshifted, while their thresholds get drastically lower than for one isolated semiconductor disk.

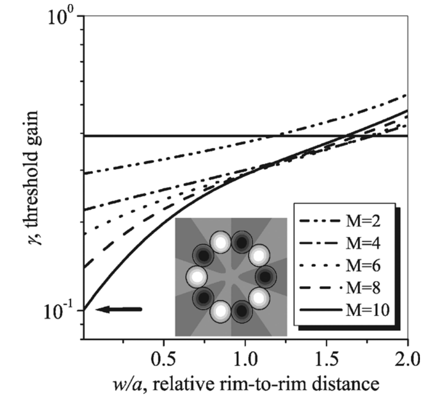


Fig. 1. Threshold values of material gain of the $H_z$-polarized supermodes $\pi^+(H_{01})_M$ versus the normalized distance between adjacent cavities. Straight solid line corresponds to the threshold value in a single microdisk. The inset illustrates the supermode near field for $M = 10$ and $w = 0.01a$.

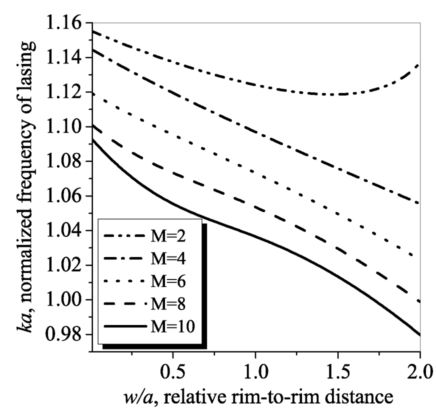


Fig. 2. Normalized frequencies of the $H_z$-polarized supermodes $\pi^+(H_{01})_M$ versus the normalized distance between adjacent cavities.

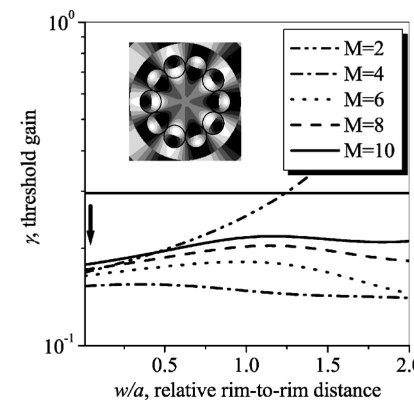


Fig. 3. The same as in Fig. 1 for the supermodes $\pi^+(H_{01})_M$.

Here, the greater the number of elementary disks in CPM, the lower the supermode threshold and the larger its frequency shift from the one-disk value. In Figs. 3–6, we present similar dependences for the $\pi^{\pm}(H_{1,1})_M$ supermodes in the same CPM. One can see that the effect of (even stronger) threshold reduction in a tightly bound CPM is observed only for the supermode $\pi^-(H_{1,1})_M$, which has maximally antisymmetric field. Its sister-mode $\pi^+(H_{1,1})_M$ maintains thresholds close to the one-disk value.

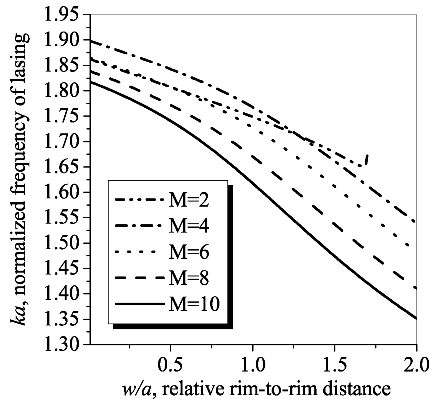


Fig. 4. The same as in Fig. 2 for the supermodes $\pi^{+}(H_{11})_{M}$.

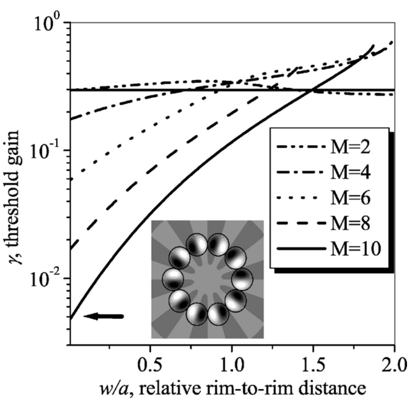


Fig. 5. The same as in Figs. 1–3 for the supermodes $\pi^{-}(H_{11})_{M}$.

Near fields of the coupled modes in CPMs can vary greatly depending on the supermode type and separation between the disks. The insets in the figures illustrate the supermodes $\pi^{+}(H_{0,1})_{10}$ and $\pi^{+}(H_{1,1})_{10}$ in tightly bound ten-disk CPMs ($w = 0.01a$). Note that the high-threshold mode $\pi^{+}(H_{1,1})_{10}$ displays visibly stronger leakage of the field out of the cavities than the other two modes whose fields are locked inside the disks.

## V. Conclusion

We have shown that linear lasing thresholds of the $\pi$-type supermodes in a CPM with the lowest order non-WG modes in

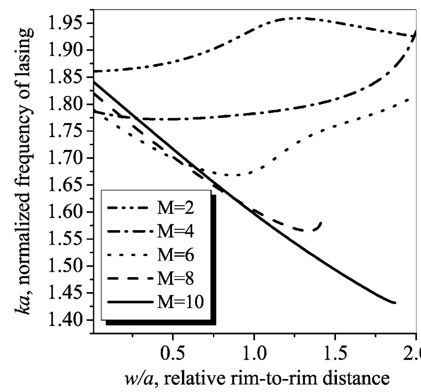


Fig. 6. The same as in Figs. 2–4 for the supermodes $\pi^{-}(H_{11})_{M}$.

each submicron size semiconductor disk cavity can be greatly reduced by 1) bringing cavities together and 2) increasing the (always even) number of cavities. We believe that this may lead to development of the "optical magnetron"—a low-threshold non-WG-mode CPM microcavity laser.

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