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Cold-Cavity Lasing Spectra and Thresholds of a Twin Disk Photonic Molecule with Optically Coupled Whispering Gallery Modes

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***Abstract*-Photonic molecule composed of two identical active microdisks is considered. Cold cavity spectra and thresholds are extracted from eigenvalue analysis showing that the latter can be lower than for a single microdisk.**

I. INTRODUCTION

Microdisk lasers were first demonstrated in the 1990's as extremely compact sources of light [1-3]. Lasing in disks of 1-10 μm diameter and 100-200 nm thickness, containing layers of quantum wells, boxes, and dots was achieved both with photopump and injection of current. The main features of such lasers are (i) periodically spaced frequencies of lasing, (ii) ultralow thresholds, and (iii) predominantly in-plane light emission. The disk modes were identified as quasi-whispering–gallery (WG) ones confined at the rim due to almost total internal reflection. Note that 3-D problem for a thinner-than-wavelength disk can be approximately reduced to the 2-D one in the disk plane, with the effective-index approach [1-3]. Description of the WG modes in the isolated disk has been done with many techniques ranging from a WKB analytical study to the FDTD numerical codes [4,5].

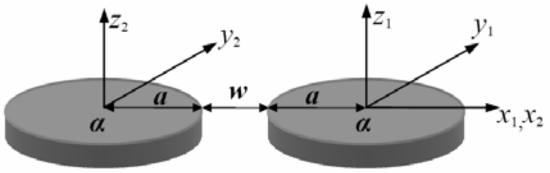

Fig. 1. 3-D geometry of two optically coupled circular disk semiconductor microresonators.

Previously, cold-cavity modeling of microlasers has been done by calculating the natural modes of the *passive* open dielectric resonators. However, it is easy to see that in this way the lasing phenomenon is not addressed directly - specific value of threshold gain needed to force a mode to become lasing is not included in the formulation.

Therefore in [6] we proposed a *Lasing Eigenvalue Problem* (LEP), specifically tailored to extract not only frequencies but also threshold gains from the field equations. Refining this analysis, in [7] we accurately accounted for the effective index dispersion and studied the effect of the gain non-uniformity on the thresholds in single microdisk.

Presently great attention is paid to the manufacturing and studying of microcavity laser arrays [8] and photonic molecules [9]. Recent experiment with two microspherical resonators showed the splitting of the coupled WG modes into symmetry classes that may have quite different lasing properties [9]. Therefore we shall study here the LEP for a twin microdisk photonic molecule supporting the WG modes.

II. 2-D LASING PROBLEM FOR TWO MICROCAVITIES

Fig. 1 shows the geometry of two identical microdisk cavities located in the same plane in free space. Suppose that each disk has thickness d, radius a, and real-valued refractive index α. Separation between the disks is denoted as *w*. Time dependence is implied as $e^{-i\omega t}$, and free-space wavenumber is $k = \omega / c = 2\pi/\lambda$, where λ is wavelength. Assume that we have already reduced the problem to the 2-D model by using the effective-index model [7]. Then we can consider two identical circular resonators with effective refractive index α_{eff} and airgap *w*.

Here, we can treat either of two polarization states separately, with the aid of one function *U*, which is either E_z or H_z field component. The LEP statement implies (see [6,7]) that *U* must satisfy 2-D Helmholtz equation, where, if $r < a$, the refractive index α_{eff} is replaced with the complex-valued parameter $\nu = \alpha^{H,E}_{eff(q)} - i\gamma$, otherwise α_{eff} =1. Here, the effective index is associated with the *q*-th guided wave of a slab of the same thickness as the disk ($1 < \alpha^{H,E}_{eff(q)} < \alpha$) [7]. We shall assume that the material gain is uniform across the disks. At the disk rims, *L*, transparency conditions are demanded. Besides, the condition of the local power finiteness is to be satisfied. Considering the LEP, we shall look for two real numbers, $\kappa = ka$ and γ. The first of them is the normalized lasing frequency while the second is the threshold material gain. Thanks to the real *k*, *U* must obey the Sommerfeld radiation condition (no divergence at infinity).

The basic properties of the lasing eigenvalues can be established even before the computations, for arbitrary-shape open resonator. It is found that all $\gamma > 0$ and

- eigenvalues form a discrete set on the plane (k, γ),
- each eigenvalue depends on resonator shape *L*, separation *w*, and refractive index α in piece-continuous

manner; this property is lost only if eigenvalues coalesce,

- eigenvalues can disappear only at infinity on (k,γ).

III. EFFECT OF COUPLING ON SPECTRA AND THRESHOLDS

The geometry in Fig. 1 has two lines of symmetry that are the *x* and *y*-axes. Therefore from general considerations it is clear that all possible field functions split to four different independent classes of symmetry with respect to these axes. By using series expansions in local coordinates and addition theorems for cylindrical functions, we reduce the problem, for each symmetry class, to infinite-matrix determinant equation with favorable features. Here, convergence to eigenvalues of infinite matrices is guaranteed if truncation number is taken greater. Practical accuracy of 4-5 digits is achieved with a few more equations than resonator's optical size, $ka\alpha$.

We computed lasing spectra and thresholds with a two-parameter secant-type iterative method [7] taking $\alpha_{eff} = const$ when computing a specific optical mode. If, e.g., $\lambda_0 = 1.55$ μm and $d = 100$ nm, then $\alpha^{H}_{eff(0)} = 2.63$ (E_z-polarized modes can be neglected in view of very high thresholds [6,7]).

Lasing modes of an *isolated circular cavity* split, thanks to the symmetry, into independent families according to the azimuth index, *m*, and those with $m > 0$ are twice degenerate [6]. One can clearly distinguish between the non-WG modes, which have very high thresholds, $\gamma > 0.01$ and $\gamma \approx const/\kappa$, and the true WG modes. The latter have $m/\alpha_{eff} < \kappa < m$ and display drastically smaller thresholds, $\gamma \approx const\ e^{-\kappa}$ [7].

In Figs. 2 and 3, we present the dependences of lasing frequencies and thresholds on the normalized separation parameter, *w/a*, for the WG modes of the family $(H_z)_{5,1}$ of all four symmetry classes. If the separation gets smaller, then the WG modes obtain frequency shifts – two of them are almost identically redshifted and two others are blueshifted. If separation is smaller than a certain value, then the thresholds of all four modes get drastically higher than for the isolated disk. However, if *w* is comparable to λ, one can achieve a threshold that is lower than the limit one for $w \to \infty$.

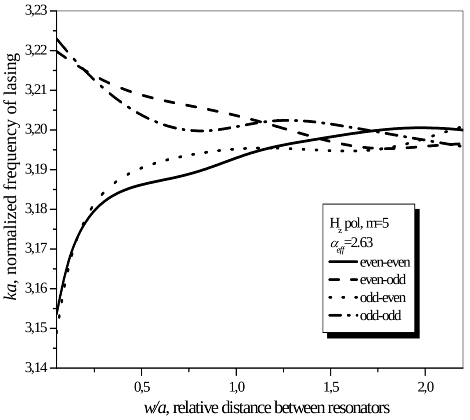

Fig. 2. Normalized lasing frequencies for the H$_z$-polarized modes of the family $(H_z)_{5,1}$, $\lambda = 1.55$, $\alpha = 3.374$ and $d/a = 0.1$.

Note that threshold reduction with respect to the single-disk case is relatively small and needs precise tuning of the separation between resonators. This reduction, in principle, may occur for the modes of any class of symmetry, however most clearly it is observed for the "twice-odd" modes.

Normalized emission patterns of the WG modes of the modes of two resonator photonic molecule display four identical main beams of emission (due to two-fold symmetry), sometimes merging into two beams along one of the symmetry axes.

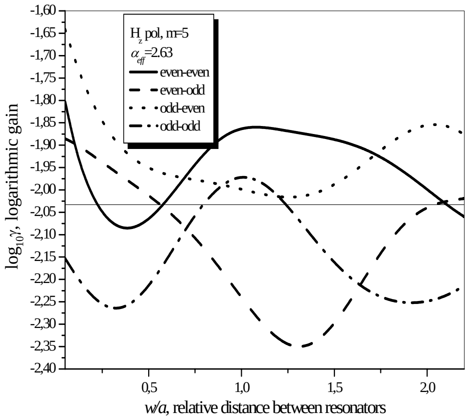

Fig. 3. Threshold gains for the H$_z$-polarized modes of the family $(H_z)_{5,1}$ in a GaAs disk, $\lambda = 1.55$ nm, $\alpha = 3.374$ and $d/a = 0.1$. Straight line is the threshold of the corresponding mode in single microcavity.

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