# Accurate Modeling of the THz Beam Scattering by a Graphene Disk

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*Abstract-* Mathematical model of the terahertz electromagnetic wave scattering by a graphene disk is based on the Maxwell equations with the resistive boundary conditions on the disk surface, where graphene electrical conductivity is included as a parameter. This problem is reduced to the Fredholm second kind integral equations (IEs) where the unknowns are the images of the normal to the disk field components in the spectral domain. A complex-source-point (CSP) field is used for the modeling of the incident to the disk wave beam.

## I. INTRODUCTION

Graphene, which is a planar hexagonal structured layer of carbon atoms, is a very promising material in nanoelectronics. Its applications in the THz range can potentially lead to new devices for the detection of explosives and CBRN agents. It is a zero bandgap semiconductor with conductivity tuned either by electrostatic or magnetostatic gating. This graphene property makes it promising for the development of ultrathin fast electronic devices [1]. To study electromagnetic wave scattering by a graphene object is necessary to couple the Maxwell's boundary value problem with phenomenological model of graphene conductivity [2]. The main challenge of such approach is to involve zero thickness of the scatterer into the model. This can be done by approximating the graphene slab with an equivalent thin layer with finite thickness. However such approach leads to the computational disaster due to very high length-to-thickness ratio (and thus very large size of matrix to be inverted). A way out of this is to model graphene as an equivalent resistive (or impedance) surface and use meshless codes for the wave scattering simulation.

In this paper we propose analytical-numerical method for the study of electromagnetic wave scattering by a thin graphene disk. We suppose that the radius of graphene disk is larger than 50 nm and thus disregard the edge effects on the graphene conductivity and use the electrical conductivity model developed for infinite graphene sheets [2,3]. To solve Maxwell boundary value problem we reduce it to the set of coupled dual IEs in the spectral domain and use the method of analytical regularization (MAR) with a Nystrom-type discretization to obtain the matrix equation.

# II. MATHEMATICAL MODEL

Consider the scattering of time-harmonic electromagnetic field by a resistive disk of radius *a*. Suppose the center of the disk is located in the origin of dimensionless cylindrical coordinates ( $\rho = r/a, \varphi, \zeta = z/a$ ). Denote the total field as a sum of the incident and scattered by the disk electromagnetic fields. Assume that the scattered field satisfies homogeneous Maxwell

equations outside the disk and the total field satisfies the following resistive type boundary conditions:

$$\left(\vec{E}_{tg}^{+}+\vec{E}_{tg}^{-}\right)=2Z_{0}\sigma^{-1}\vec{n}\times\left(\vec{H}_{tg}^{+}-\vec{H}_{tg}^{-}\right),\ \left(\vec{E}_{tg}^{+}-\vec{E}_{tg}^{-}\right)=0$$

Here  $Z_0$  is the free space impedance,  $\vec{n}$  is the normal to the disk unit vector, and  $\sigma$  is the graphene surface conductivity, which can be determined from the Kubo formalism [2].

For completeness, the total electromagnetic field must satisfy the radiation condition and the condition of local integrability of power.

## III. SOLUTION METHOD

To solve the formulated problem we use the method of dual IEs in the spectral domain together with the concept of analytical regularization [4]. That enables us to reduce the problem to a set of Fredholm second kind IEs on the semi-infinite interval for the unknown images of the normal to the disk field components. The features of such equations guarantee the existence and uniqueness of their exact solution, thus it is possible to use any reasonable analytical or numerical scheme to solve them. We use the numerical technique which is based on the truncation of the integration interval to the finite one combined with the descretization of the truncated equations using the Nystrom-type scheme. Finally we obtain a set of linear algebraic equations and solve them numerically using the Gauss inversion of the corresponding matrix. For the modelling of the wave beam which propagates along the disk axis, we use a 3-D CSP model with the source coordinate  $\zeta_0$  shifted to the complex plane.

Thus we obtain a stable numerical meshless algorithm for solving the quasioptical beam scattering by a graphene disk. We calculate the scattered and absorbed powers and the far field patterns for such a wave beam incident on the graphene disk in the THz frequency range.

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