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SPECTRAL THEORY OF PRINCIPAL AND HIGHER ORDER MODES IN OPEN CIRCULAR CYLINDRICAL SLOT AND STRIP LINES.

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Abstract.

The problem of mode spectrum in regular partially screened circular dielectric rod is analysed by means of the theory of analytical operator-functions technique. The spectrum is proved to exist, to be descrete and to consist of both principal and higher order modes, including slot and strip waves. Constants of propagation, impedance and losses of modes as functions of frequency and geometrical parameters are investigated numerically.

1. Introduction.

Electromagnetic waves guided by transmission line, cross-section of which is shown in fig.1, are investigated. This line may be viewed as a standard open waveguide of nonplanar type. When the slot is more narrow than the strip ($\theta<\delta$) it is called circular cylindrical slot line (CSL), while in the opposite case ($\theta>\delta$) - circular cylindrical microstrip line (CML).
Nonplanar lines have began to be interested in more recently than planar ones. Static or quasistatic approximation for T and quasi-T modes in lines analogous to CML were considered in [1-3], full-wave solution attempt being restricted to the case of a narrow strip only.
Here the full spectral problem is considered in correct electrodynamic formulation (see [4]). The main idea is to reduce it to determination of characteristic values of some Fredholm matrix operator-function. Numerical solution of dispersive equation obtained provides computation of propagation constants, impedance and losses with any given accuracy. The losses are calculated by means of the perturbations theory technique modified in accordance with [5].

2. Spectral problem.

Assume the electromagnetic field in the waveguide to be a normal guided wave with longtitudinal components $(E_z,H_z)=(u(\vec{r}),v(\vec{r}))e^{ihz-i\omega t}$, where h is a complex constant of propagation. To solve spectral problem means to determine those values of parameter h, which generate nontrivial solutions $\overline{W}(\vec{r})=(u,v)\in C^2(\mathbb{R}^2\setminus(r=a))\cap C^1(\mathbb{R}^2)$ of equation

$$(\Delta+\tilde{g}^2)\overline{W}(\vec{r})=0 \quad \text{for } \vec{r}\in\mathbb{R}^2\setminus(r=a) \qquad (1)$$

satisfying boundary conditions on the slot $\partial L=(r=a,|\varphi|<0)$ and the strip $\partial M=(r=a)\setminus\partial L$

$$[B_E W]^+_-|_{\partial L}=0,\ [B_H W]^+_-|_{\partial L}=0,\ B_E W|_{\partial M}=0, \qquad (2)$$

condition of local energy limitation

$$\int_D (|k\overline{W}|^2+|\mathrm{grad}\overline{W}|^2)\,r\,dr\,d\varphi<\infty \quad \text{for } D\subset\mathbb{R}^2, \qquad (3)$$

and Reichardt "radiation" condition [6] for all $r>a$

$$\overline{W}(\vec{r})=\sum_n (a_n,b_n)H_n^{(1)}(gr)e^{in\varphi} \qquad (4)$$

Here $\tilde{g}^2=k^2\varepsilon-h^2$, $g^2=k^2-h^2\neq 0$, k is a free space wavenumber and differential operators B_E, B_H follow from Maxwell equations. Complex eigenvalues of h form the spectrum σ of generalized normal eigenmodes of open waveguide. In view of (4) h is assumed to lay on Riemann surface $\mathcal{L}$ of the function $\mathrm{Ln}(k+h)(k-h)$, which has two branch points $h=\pm k$ and consists of infinite number of sheets. A note should be made that problem (1)-(4) depends on the angular width of the slot 2θ or the strip $2\delta=2(\pi-\theta)$ in a singular way. Therefore one may suppose the spectrum $\sigma(\theta)$ to have different structure at $\theta=0$, $0<\theta<\pi$ and $\theta=\pi$.

3. Equivalent operator-function.

Let us expand the components of $\overline{W}(\vec{r})$ in Fourier series satisfying (1) and (4)

$$u(\vec{r})=\sum_n z_n^E U_n(r/a)e^{in\varphi},\quad v(\vec{r})= \qquad (5)$$

$$\sum_n (xz_n^H-nq_H p_n z_n^E)f_nF_n(f_n-F_n)^{-1}V_n(r/a)e^{in\varphi}$$

where $U_n(t)$ and $V_n(t)$ are functions proportional to $J_n(yt)$ if $t<1$ or $H_n^{(1)}(xt)$ if $t>1$, $p_n=(xF_n)^{-1}$ if $r>a$ or $(xf_n)^{-1}$ if $r<a$, $x=ga$, $y=\tilde{g}a$, $f_n=J_n'(yJ_n)^{-1}$, $F_n=H_n'(xH_n)^{-1}$, $J_n=J_n(y)$ $H_n=H_n^{(1)}(x)$. After some manipulations on (5) and (2) one obtains dual series equations

$$\sum_n z_n^s e^{in\varphi}=0,\quad \varphi\in\partial S_{(s)} \qquad (6)$$

$$\sum_n z_n^s|n|e^{in\varphi}=(c_n^s z_n^s+nq_s d_n^s z_n^t)e^{in\varphi},\ \varphi\in\partial S_{(t)}$$

where $s,t=E,H$, $s\neq t$, $\partial S_{(E)}=\partial M$, $\partial S_{(H)}=\partial L$,

$$q_E=\frac{ihka^2(\varepsilon-1)}{2x(\varepsilon+1)},\quad q_H=\frac{ihka^2(\varepsilon-1)}{xy^2},$$

$$c_n^E=|n|-\frac{(x^2+y^2)D_n}{2(\varepsilon+1)(f_n-F_n)},\quad c_n^H=|n|+\frac{(x^2+y^2)f_nF_n}{f_n-F_n}$$

$$d_n^E=1+\frac{(x^2+y^2)F_n}{y^2(f_n-F_n)},\quad d_n^H=1-\frac{(x^2+y^2)f_n}{x^2(f_n-F_n)},$$

$D_n=(\varepsilon f_n-F_n)(f_n-F_n)-(nkh)^2a^4(\varepsilon-1)^2x^{-2}y^{-2}$, with the condition following from (3)

$$\sum_n |z_n^s|^2|n+1|<\infty,\quad s=E,H \qquad (7)$$

Left-hand side of (6) forms Riemann-Hilbert boundary value problem, solution of which leads to linear algebraic equation generating the operator equation

$$[I - A(h)]Z = 0, \qquad (8)$$

where

$$Z=(z^E_n, z^H_n)^{\infty}_{n=-\infty}, \quad A(h)=\|(A^{ij}_{mn})^{\infty}_{m,n=-\infty}\|_{i,j=1,2}$$

$$A^{11}_{mn}=c^E_n T_{mn}(u), \quad A^{22}_{mn}=(-1)^{m+n} c^H_n T_{mn}(-u),$$

$$A^{12}_{mn}=nq_E d^E_n T_{mn}(u), \quad A^{21}_{mn}=(-1)^{m+n} nq_H d^H_n T_{mn}(-u),$$

$$T_{oo}(\pm u)=-\ln((1\mp u)/2), \quad T_{on}=(P_{n-1}-P_n)/(2n),$$

$$T_{mn}=(P_{m-1}P_n-P_mP_{n-1})/(2m-2n), \quad P_n=P_n(\pm u)$$

are Legendre polynomials, $u=\cos\theta$.

Basing on the estimates of A^{ij}_{mn} as $|m|,|n|\to\infty$ one may show $A(h)$ to be a compact operator in Hilbert space of pairs of number sequences $l^2_2=l_2 \times l_2$. Consequently (8) is a Fredholm equation, solution of which may be shown to generate functions (5) of the needed class. Therefore problem of characteristic values of (8) is spectrally equivalent to (1)-(4) on $\mathcal{L}\setminus\Lambda$, where the set Λ consists of the points $h=\pm k$, $h=\pm h_\varepsilon=\pm k[(\varepsilon+1)/2]^{1/2}$ and poles of $c^{E,H}_n$, $d^{E,H}_n$.

4.Fundamental properties of spectrum.

Detailed investigation shows $A(h): l^2_2 \to l^2_2$ to be a finite meromorphic operator-function on $\mathcal{L}\setminus(\pm k, \pm h_\varepsilon, \infty)$. Going from energy theorems one may prove the existence of nonempty free of spectrum domain $\rho\subset\mathcal{L}$ where $I - A(h)$ is invertible. When $\mathrm{Im}\, k>0$ and (or) $\mathrm{Im}\,\varepsilon>0$ ρ includes the whole real axe of "physical" sheet of $\mathcal{L}$ (no proper eigenmodes exist). When $\mathrm{Im}\, k=0$ and $\mathrm{Im}\,\varepsilon=0$ ρ includes the parts of this axe external to the intervals $k<|h|<k\sqrt{\varepsilon}$. Basing on this and theorems from [7], one comes to

Theorem 1. σ is at most a descrete set of points of finite multiplicity with the only accumulation point at infinity on $\mathcal{L}$. Proper eigenmodes exist if $\mathrm{Im}\, k=0$ and $\mathrm{Im}\,\varepsilon=0$ only and their number is at most finite.

Spectrum symmetries on $\mathcal{L}$ are proved by substitution to (1)-(4). σ consists of pairs $(\pm h)$ or quartets of points $(\pm h, g)$, $(\pm h^*, -g^*)$ provided that $\mathrm{Im}\, k=0$ and $\mathrm{Im}\,\varepsilon=0$.

In order to prove the existence of σ one may use the theorems from [7] and the fact that it is true for some more plain structures: closed waveguide and its external domain ($\theta=0$) and circular dielectric rod ($\theta=\pi$). Here the main difficulty is caused by noncontinuousness of $A(h,\theta)$ on θ as $\theta\to 0$ and $\theta\to\pi$.

Manipulating on (8) one obtains two independent equations jointly equivalent to (8)

$$[I - A^{(\pm)}(h,\theta)]Z^{(\pm)} = 0, \qquad (9)$$

where

$$Z^{(\pm)}=(Z^{E\pm}, Z^{H\pm}), \quad Z^{s\pm}=(z^s_n \pm z^s_{-n})^{\infty}_{n=0}, \quad s=E,H,$$

$$A^{ij(\pm)}=(A^{ij}_{mn} \mp (-1)^i A^{ij}_{-mn})^{\infty}_{m,n=0}, \quad i,j=1,2$$

It means that σ splits into two sets $\sigma^{(\pm)}$ generating two orthogonal families of waves: $E_z$odd/$H_z$even and $E_z$even/$H_z$odd in the sence of symmetries to the plane $\varphi=0,\pi$.

Operator-function $A^{(+)}(h,\theta)$ depends continuously on $\theta\in[0,\pi)$ while $A^{(-)}(h,\theta)$ on $\theta\in(0,\pi]$, that gives way to prove

Theorem 2. σ is not empty for all $0\leq\theta\leq\pi$. Points of σ depend continuously on θ and analytically on ka and ε if nonspectral parameter is not a value of their coalescence.

5. Singular and principal modes.

It has been found out that for any slot width ($\theta\neq 0$) $\sigma^{(-)}$ includes a point generating so-called slot mode given by

$$h(H_{oo})=k[(\varepsilon+1)/2+(ka)^{-2}\ln^{-2}\sin\tfrac{\theta}{2}]^{1/2} \qquad (10)$$

As $\theta\to 0$ slot mode is of quasi-H type viewing from the point of field structure. It has a small but finite cutoff frequency, that is why it is not a principal mode. In a similar way for any strip width($\delta\neq 0$) $\sigma^{(+)}$ includes a point generating so-called strip mode given by

$$h(E_{oo})=k[1+2(\varepsilon-1)(4+\frac{\ln(\varkappa^2/\varepsilon+1)}{\ln\sin(\delta/2)})^{-1}]^{1/2}, \qquad (11)$$

where $\varkappa=ka(\varepsilon-1)^{1/2}$.

At contrast to slot mode strip one is not only a quasi-E mode as $\delta\to 0$ but also a principal (quasi-T) mode with no cutoff frequency. Besides, CML spectrum includes two more principal modes, namely, $H\mp E^{\pm}_{11}$ modes of dielectric rod perturbed by the strip. As for CSL spectrum, it includes not only slot mode and other higher order ones but also three principal modes: quasi-T_o and quasi-$T^{\pm}_{1}$. Slot mode and quasi-$T^{\pm}_{1}$ modes are singular ones of CSL while strip mode is a singular one of CML because they disappear from $\sigma^{(+)}$ or $\sigma^{(-)}$ when $\theta=0$ or $\delta=0$. All the other modes transform continuously to E and H-modes of closed waveguide and its external domain as $\theta\to 0$ and to modes of open circular rod as $\delta\to 0$.

6. Numerical results.

As $A^{(\pm)}(h)$ are compact in l^2_2 the points of $\sigma^{(\pm)}$ may be appoximated by characteristic values of truncated equations (9) or the zeros of their determinants. Analyticity of A(h) guarantees possibility of highly effective iterative algorithm to be applied basing on Newton principle.

Fig.1 shows transformation of CSL modes to CML ones with widening the slot. Change of azimuthal index during the transformation points out to coupling effect with the other mode at some $0<\theta<\pi$.

Fig.2 demonstrates evolution of mode fields with variation of θ.

Slot and strip mode impedances are given by

$$Z(H_{oo})=V^2/2P, \quad Z(E_{oo})=2P/I^2, \qquad (12)$$

where

$$V=a\left|\int_{\partial L} E_\varphi(a,\varphi)d\varphi\right|, \quad I=(4\pi a/c)\left|\int_{\partial M}[H_\varphi(a,\varphi)]^+_- d\varphi\right|,$$

$$P=(c/8\pi)\iint_{R^2}(E_rH^*_\varphi-E_\varphi H^*_r)r\,dr\,d\varphi$$

and the results of computations are presented in fig.3.

Losses in dielectric and metal are given by

$$\alpha_d = (ck\,\mathrm{Im}\,\varepsilon / 8\pi P) \int_0^a \int_0^{2\pi} |\vec{E}|^2 r\,dr\,d\varphi \tag{13}$$

$$\alpha_m = (ckda/32\pi P) \int_{-\delta+\Delta}^{\delta-\Delta} (|\vec{H}^+|^2 + |\vec{H}^-|^2)\,d\varphi \tag{14}$$

where $\Delta = c^L w/a$, $c^L = 1/2\pi e$ is Lewine constant [5], $d = (\lambda c/\tau)^{1/2}(2\pi)^{-1}$ is the skin-layer depth, τ is conductivity, 2w is the thickness of the strip with rectangular edge. Fig.4 and 5 show that α_m is several times greater than α_d provided the wavelength is not too short. The curves in fig.3 and 5 fall down to zero at cutoff frequency (when slot mode becomes a leaky one) due to unlimited growth of P.

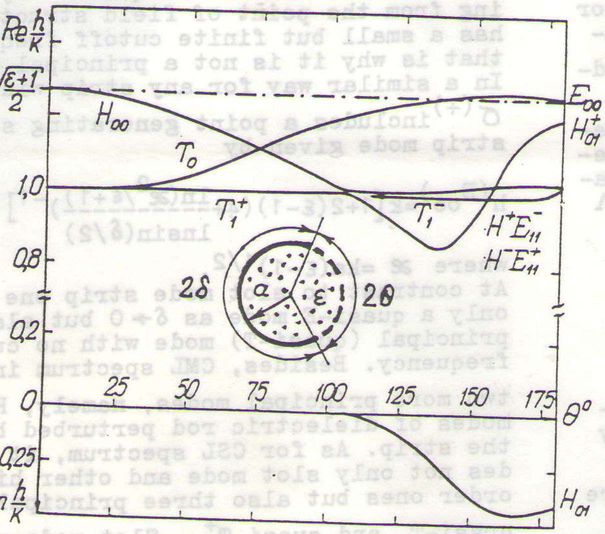

Fig.1 Transformation of modes with variation of θ. ka=1.25, ε=2.25.

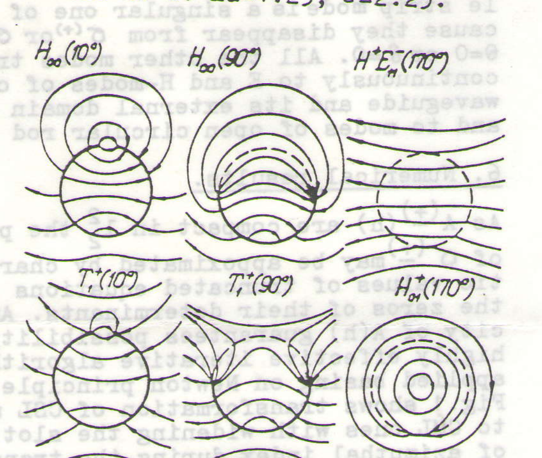

Fig.2 E-field strength lines evolution in the cross-section. ka=1.25, ε=2.25.

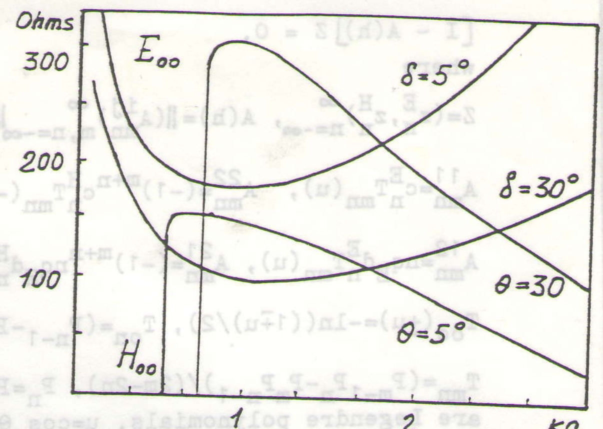

Fig.3 Slot and strip mode impedance variation with normalized frequency ka for different θ and δ ε=2.25.

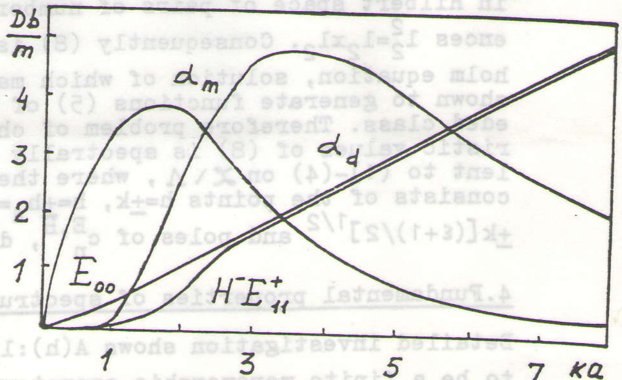

Fig.4 Strip line losses variation with frequency. ε=2.25, a=1mm, 2w=0.1mm, $\delta=5^o$, $\tau=5{\cdot}10^{17}s^{-1}$, $\mathrm{Im}\,\varepsilon=2.25{\cdot}10^{-4}$.

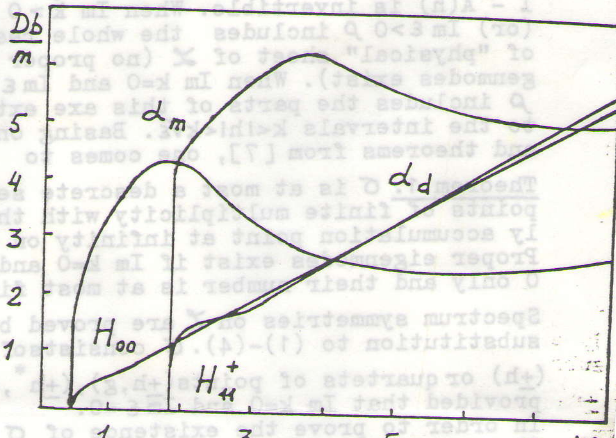

Fig.5 Slot line losses variation with frequency. ε=2.25, a=1mm, 2w=0.1mm, $\theta=5^o$, $\tau=5{\cdot}10^{17}s^{-1}$, $\mathrm{Im}\,\varepsilon=2.25{\cdot}10^{-4}$.

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