

## On the Effect of Cross-Sectional Compactness on Mode Behaviour in Microstrip and Slot Lines

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### ABSTRACT

The problem is concerned of wave propagation along the uniform microstrip and slot lines on circular dielectric substrates. Results obtained by a highly effective numerical solution give the way for better understanding of modes behaviour in the lines with laterally compact cross-section modelling real printed-circuit waveguides. It is shown that no dominant mode leakage is observed as the frequency of operation is increasing.

### INTRODUCTION

Although planar microstrip and slot lines are well-known as the models of printed microwave guides, their circular and other non-planar counterparts have begun to be interested in rather recently. These lines may be prospective as flexible guides and miniature antennas in microwave applications. Besides, taking into account that in practice the frequency of operation is pushing higher, the circular lines can turn to be more reliable models of real printed-circuit waveguides on finite (compact) substrates.

Zeng and Wang in [1] and Alexopoulos and Nakatani in [2,3] have considered the dominant principal modes on circular cylindrical substrate around the central metallic core. Here we consider open waveguides of similar geometry, but central conductor is absent (Fig. 1)

of MS lines

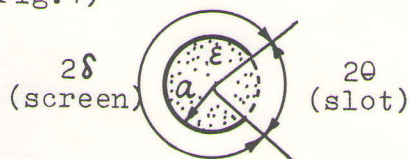


Fig. 1 Cross-sectional view of a guide under consideration strip-line for  $\delta < \theta$ , slot-line for  $\theta < \delta$ .

As has been reported in [4], the circular shape of cross-section enables one to apply a powerful mathematical method of treatment and obtain a highly effective numerical algorithm. In contrast to [1-3], it is based on the full-wave dynamic solution. The results presented cover both principal and higher order modes on strip and slot lines with arbitrary strip or slot widths.

### FORMULATION AND TREATMENT OF MODE SPECTRAL PROBLEM

Assuming the field to be a normal guided mode with a complex constant of propagation  $h$ , one comes to 2-D cross-sectional eigenvalue (spectral) problem [4]. Due to the circular shape of boundaries, fields inside and outside the dielectric are expandable in terms of cylindrical wave functions. This leads to dual series equations with a kernel of "trigonometric" type for expansion coefficients. By using so-called Riemann-Hilbert problem technique, dual equations are reduced to infinite system of linear algebraic equations. Mathematical study of corresponding matrix operator proves that it is of a Fredholm 2-nd kind type. This fact guarantees both rapid convergence and any desired accuracy in seeking complex zeros of the deter-

minant provided that the order of truncation is sufficiently large.

Based on this treatment, one may also compute field structure and impedance of the mode under investigation as well as the losses in dielectric core. As for the losses in metal screen, their correct calculation needs to take into account finite thickness of real conductor.

#### PRINCIPAL AND HIGHER ORDER MODES BEHAVIOUR

Conventional models of open printed-circuit lines usually contain laterally infinite metal conductors or/and dielectric substrates. Recently it has been discovered that in these lines the dominant surface mode turns into leaky one provided that frequency has exceeded certain fixed value. This phenomenon was discussed in details by Shigesawa et al. in [5], who had reported about some new features of it for strip lines on anisotropic substrates.

The physical mechanism that produces this leakage is rather obvious. If phase constant of a mode on a strip or slot line  $h$  becomes less than the same quantity  $g$  of the principal guided mode on surrounding dielectric substrate layer (with or without conductor backing), then far from the strip or slot in lateral direction the field leaks at an angle  $\theta$ :  $\cos\theta = h/g$ , while remaining nonradiative in normal direction. In other words, the condition  $h=g$  makes a threshold of opening or closing new channel of power leakage to infinity in the cross-section, other than the free-space radiation. From this viewpoint it is clear that for the models of open transmission lines on finite substrates the situation is quite different. There are no channels of leakage here other than free-space radiation. That is why neither dominant nor any other surface mode is expected to have high frequency cutoff until the substrate remains to be finite.

However, it seems that essentially no reliable numerical information is available for open transmission lines on compact substrates. Circular ones give a lucky chance to obtain it accurately following the approach described above.

Figs. 2 and 3 show the results of computations of normalized phase

and leakage constants for modes on circular lines with different strips or slots. No leakage is observed for any frequencies above low-frequency surface wave cutoff, and the modes remain to be purely bounded.

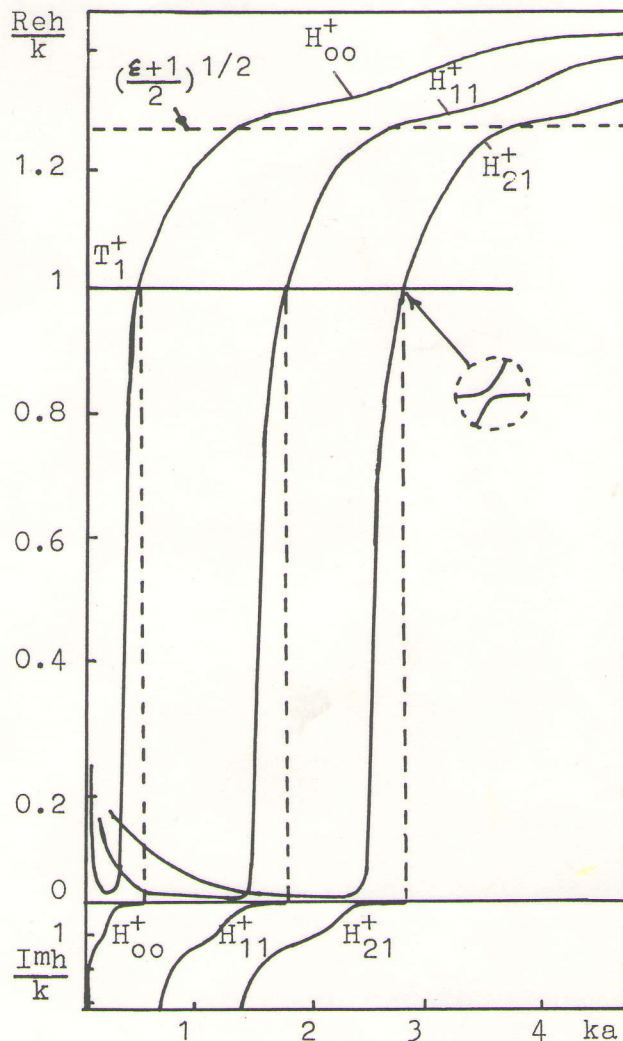


Fig.2 Dispersion of  $E_z^-/H_z^+$  modes on a slot line with  $\epsilon=2.25$ ,  $\theta=0.1^\circ$ .

Besides, dispersion curves show that there exist some values of frequency for which the curves come close one to another, pointing out to the effect of mode "coupling" between the principal and e.g. higher order modes. The narrower the strip or the slot, the more close coupling is observed. A comment should be made that there are two orthogonal mode families in circular lines with different field symmetries in the cross-section: one may be designated as  $E_z^+/H_z^-$  while the other as  $E_z^-/H_z^+$ . The coupling mentioned takes place within the same family of modes.

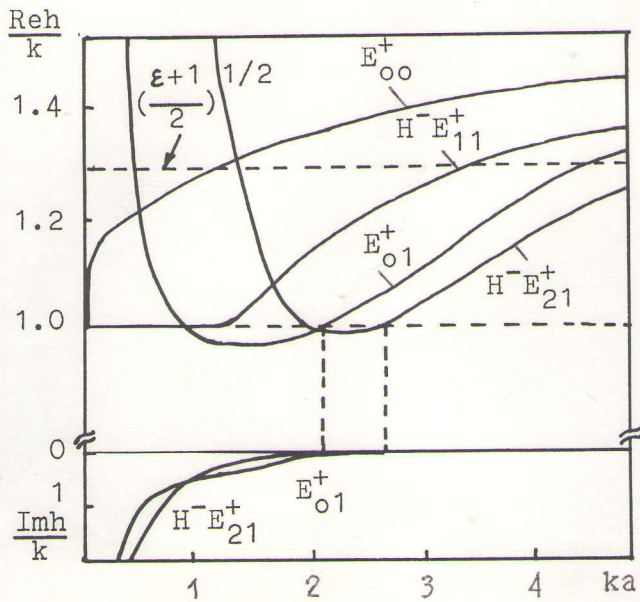


Fig. 3 Dispersion of  $E_z^+/H_z^-$  modes on a strip line with  $\epsilon = 2.25$ ,  $\delta = 1^\circ$ .

It is worth to emphasize that open metal-dielectric lines can never operate as purely single-mode guides because of several surface modes propagating without low-frequency cutoff. These modes are usually called principal ones after Sommerfeld, who had discovered them firstly on a conducting wire. As may be shown, their number is equal to  $N+2$ , where  $N$  is the number of conductors. Thus the lines presented in Fig. 1 support three principal modes: one of them is former TEM wave of a screen (a single wire line) slowed by dielectric rod while two others are principal modes of a rod, perturbed by a screen.

Nevertheless, if one of guided modes has significantly greater value of phase constant than all the others, it is called a dominant one, and the line may be viewed as a quasi-single-mode guide.

Analytical treatment, as well as numerical computations, show that on circular lines having narrow strip or slot the dominant modes are of different symmetry families. On strip line this is a "strip" mode, or quasi- $E_{00}^+$ , which is a principal one, while on slot line this is a "slot" mode, or quasi- $H_{00}^-$ , which has extremely low but finite cutoff frequency. The latter tends to zero with narrowing the slot as  $O(\ln^{-1}\theta)$ . Below the cutoff slot mode turns into a leaky wave radiating

to surrounding free space just in the same manner as any other higher order mode.

Fig. 4 shows the evolution of slot-line modes to strip-line ones. Both quasi- $E_{00}^+$  and quasi- $H_{00}^-$  modes are of a singular nature, as the former does not exist on a single dielectric core, while the latter does not exist in a closed waveguide. With narrowing the strip or the slot phase constants of these two modes tend to the same value  $k((\epsilon+1)/2)^{1/2}$ . Circles and triangles in Fig. 4 represent experimental verification of this evolution. Disagreement with theoretical curves is due to the finite thickness of real conducting screen. Note that the mode coupling effect takes place again, although its explanation is not so straightforward as for curves in Figs. 2, 3.

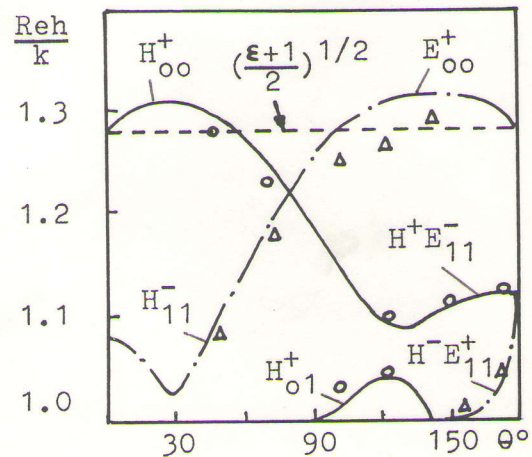


Fig. 4 Evolution from slot-line modes to strip-line ones on substrate with  $\epsilon = 2.25$ ,  $ka = 1.76$ .

Seeking the eigenfunction of initial boundary value problem corresponding to the eigenvalue of  $h$ , one obtains field structure of the mode. Fig. 5 gives transverse E-field pictures for strip and slot modes on the small diameter lines.

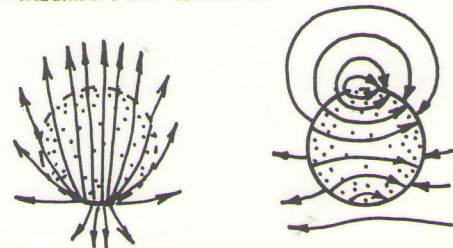
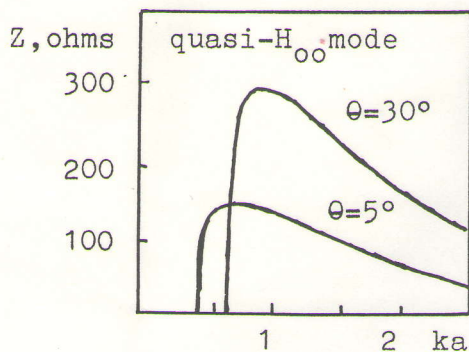
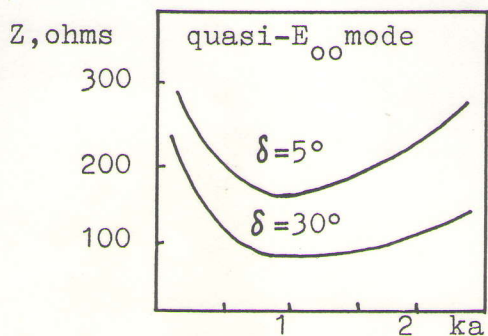


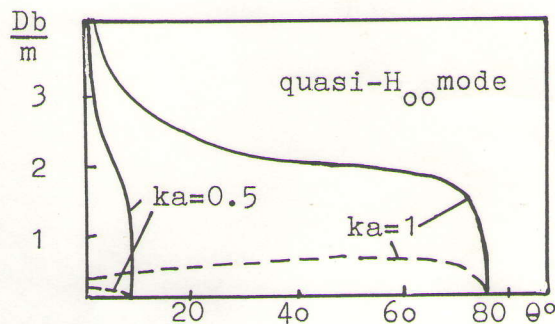
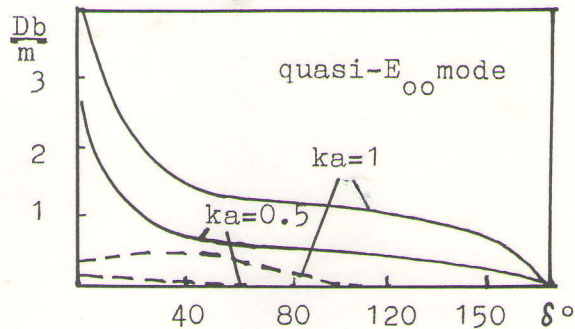
Fig. 5 Transverse E-field structure for strip mode ( $\delta = 10^\circ$ ) and slot one ( $\theta = 10^\circ$ ).  $\epsilon = 2.25$ ,  $ka = 1.5$ .

After finding the fields, it is possible to calculate mode characteristics needed for microwave applications. For dominant modes one may introduce impedances in a conventional way through the longitudinal current over the strip or the voltage over the slot related to the total cross-sectional power of the mode. The results of computations are presented in Figs.6,7. When comparing them with similar quantities for another transmission lines one should remember that strip and slot mode impedances have different definitions.



Figs.6,7. Strip- and slot-mode impedances versus normalized frequency for  $\epsilon = 2.25$ .

Losses in dielectric core are proportional to the fraction of power propagating inside the core, while losses in metal screen are caused by the currents on inner and outer surfaces of the screen (see [4]). With narrowing the slot or the strip the former tend to a constant while the latter grow infinitely, as in Figs.8,9. Losses dependences versus frequency show a linear growth of dielectric losses for both types of lines. As for losses in metal, they grow less rapidly and even decrease for strip line due to field concentration inside the dielectric substrate.



Figs.8,9. Strip- and slot-mode losses in metal (solid line) and dielectric (dashed line) for substrate/screen parameters  $a=1\text{mm}$ ,  $w=0.1\text{mm}$  (thickness),  $\sigma=5 \cdot 10^{17}\text{s}^{-1}$  (conductivity),  $\epsilon=2.25(1+i \cdot 10^{-4})$ .

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