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Microwave Analogue of Tamm States in Periodic Chain-Like Structures

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Abstract—The paper is devoted to the study of microwave analogue of Tamm states appearing at the boundary of two different periodical chain-like structures in contact. A comparison of numerical and experimental data is provided for periodical chains of quadripoles modelling our system. As it turns out, at the point of contact of two different periodic structures, significant concentration of electromagnetic wave energy takes place. The corresponding concentration of energy is quite similar to those characteristics for Tamm states concentration which takes place at the boundary between two adjacent photonic crystals. We use the term microwave analogue of Tamm states for the considered periodic chain-like structures.

1. INTRODUCTION

Surface electronic Tamm states are known to appear at a flat boundary of semi-infinite crystal with vacuum. Characteristic geometry, which is responsible for appearance of the Tamm states, includes, on the one hand, the presence of barrier potential at the interface with vacuum, preventing electrons to escape from the metal, and on the other hand, the presence of periodic lattice, with its permitted and forbidden zones of electron energies. For initiation of surface Tamm states as a necessary condition the electron energy should lie just in a forbidden zone of crystal lattice [1, 2].

The recent advent of macroscopic periodic structures, photonic and magnetophotonic crystals, allows, though only for electromagnetic waves, the reproduction of characteristic geometry features, which brings about the appearance of surface Tamm states [3, 4]. Really, on one side we have a periodic medium for propagation of electromagnetic waves. In such a medium, a characteristic band structure is known to exist for these waves. At the boundary of such a periodic medium, we place a medium in which electromagnetic waves decay rapidly. In particular, it may be a medium with negative permittivity or permeability. The negativity of permittivity takes place below the electron plasma frequency (the Drude formula) as the negativity of permeability taking place in the vicinity of the ferromagnetic resonance [5, 6].

Now studies of different surface states attract attention of numerous research groups. Some of these states are named Tamm states by analogy with surface electronic Tamm states. Recent papers [7–11] contain information about attempts to create the polariton laser with mechanism of action based on the emission of coherent and monochromatic light with exciton-polaron Bose-Einstein condensate. The use of superlight particles makes it possible to increase the temperature of Bose condensation up to the room temperatures. Surface states playing a special role in the process are named now Tamm plasmon-polariton (optical Tamm) states. Note that these optical Tamm states cannot form on the crystal surface but only on the interface of two periodic dielectric structures with different periods.

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A monographic review of electronic surface Tamm states was published in 1970 [12]. A detailed review for surface electromagnetic waves in one-dimensional periodic structure — photonic crystal — can be seen, e.g., [4–8]. In particular, a special feature of the surface Tamm states was emphasized there, i.e., the surface state appears when the wave vector of falling electromagnetic wave is normal to the interface boundary which separates the periodic medium from the medium with rapid attenuation of the wave. The transmission coefficients separately for each of the media (the electromagnetic wave frequency lays in a forbidden zone) are much lower than the transmission coefficient through the combined system. In this case, we can say that narrow Tamm peak of transmission coefficient for the combined structure is connected with tunnelling of electromagnetic wave through the Tamm state. Note that according to [4, 7], Tamm states can also occur at the boundary of two adjacent photonic crystals in the frequency range corresponding to the coincidence of band gaps of these two crystals.

The purpose of this paper is to study the analogue of Tamm states arising at the boundary of two different periodical chain-like structures in contact. We consider here some kind of chain-like structure, bearing in mind its possible future applicability to description of wave propagation in similar nanostructured systems (nanowires, set of quantum junctions etc.). Because of geometry of our problem, we cannot talk about the surface Tamm states. However, as it turns out, at the point of contact of two different chain-like structures, a significant concentration of electromagnetic wave energy may take place. We stress here that the necessary condition of occurrence of this phenomenon is the coincidence of forbidden zones. In other words, the sufficient condition is equality of subsystems' impedances (the matching of whole system) at certain frequency (Tamm frequency) in this band.

The corresponding concentration of energy is quite similar to those characteristics for Tamm state concentration which takes place at the boundary between two adjacent photonic crystals. Thus, we use the term microwave analogue of Tamm states for the considered periodic chain-like structures.

2. EXPERIMENTAL AND SIMULATION DETAILS

As a model of our chain-like structure, we use a line which represents a sequence of quadrupoles [13, 14]. The structure is a periodic one, namely, it represents a periodic repetition of certain basic "cell" consisting of quadrupoles.

To concretize our calculations and to simplify the comparison with experiment, our elementary basic quadrupole cells will be considered below based on two-port microstrip structures. Note that the use of such microstrip structures in the microwaves is a widespread technique now, which made it possible not only to obtain a number of approximate formulas to describe their physical properties, such for example as the resistance [13], but also to develop methods for production of complex microstrip structures with photonic band gap. These structures are direct analogues of known photonic crystals (see, for example, [14]).

For calculations of transmission coefficient through the periodic structure, the transfer matrix method was used. The transfer matrix is known to possess an important property of multiplicativity: the transfer matrix for a composite system is the product of the transfer matrix of its constituent elements. Using expressions of transfer matrix elements for elementary cells and the periodicity of our structure, we obtain at last the Bloch equation for infinite periodic chain. For details of calculations see [15].

Obtained in [15] Bloch equation determines the Bloch wave vector k_B. When for periodic infinite chain of quadripoles k_B is a pure imaginary number (or a complex number for finite chain), electromagnetic fields proportional to $\exp(ik_B z)$ attenuate in such a medium, and the penetration of electromagnetic wave into the medium is impossible (for the second case the wave strongly attenuates in the medium). The corresponding frequency band in the spectrum of waves is so-called forbidden band. For pure real k_B, we are dealing with a pass band of the crystal — the electromagnetic wave propagates freely through the medium (here we consider a medium without attenuation). The existence of pass and forbidden bands in the frequency spectrum is the inherent property of any periodic superlattice [16].

As shown in details in [15] (see Section 3, Figures 4(a), (b)), the results of numerical solutions of Bloch equation for the pass and forbidden bands in the frequency spectrum of periodic quadripole chain, which models our system, give a sufficiently good picture of forbidden and pass bands already for small number of elements $N = 8$.

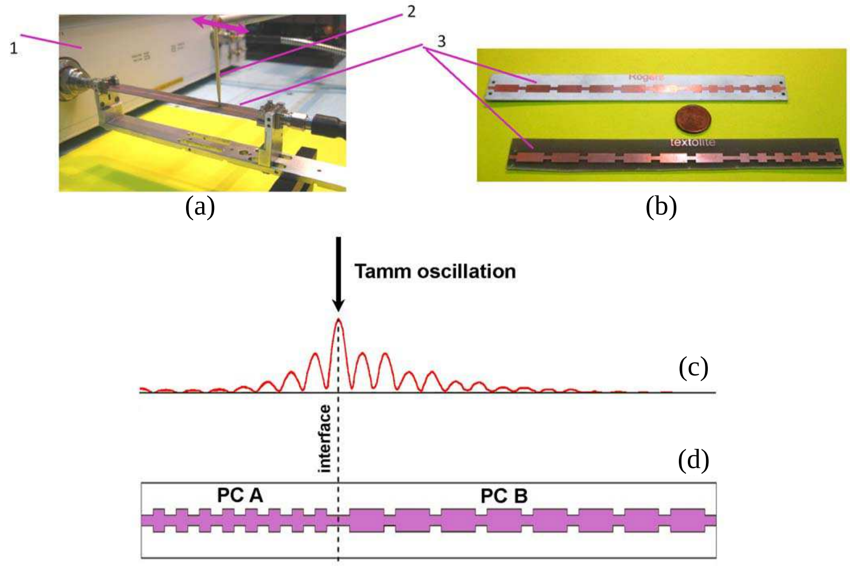

Figure 1. (Color online) (a) The setup used for experimental study of field distribution; (b) the composite structure under study; (c) the spatial field distribution (numerical results), (d) for our model system consisting of two periodic subsystems with 8 elements in each.

The numerical calculation of transmission coefficient through the system and the corresponding experimental test was carried out for the line, consisting of two chain-like periodic subsystems (PC A and PC B) [15]. Each experimental subsystem contains 6 elements with different parameters (the length of sections), see Figure 1(b). An important feature of the whole line which models our system is its matching. The condition of matching is known, to be the equality of impedances of line subsystems [13] (see also [14], Chapter 1, (12)).

A photo of experimental setup used to study a field distribution in the system is presented in Figure 1(a). The setup includes the Vector Analyzer Agilent PNA-L N5230A (1), microstrip line (3) with modulated width of central stripe (PC A or PC B), and a scanning device which moves the holder with an absorbing test probe (2) along the microstrip line (3) (composite periodic structure (PC A or PC B) with various lengths of subsystems elements (see inset (b)). Experiment is carried out for the following subsystems parameters: the width of narrow elements $w_N = 0.5\,\mathrm{mm}$, wider elements have $w_W = 3\,\mathrm{mm}$, and the length of narrow elements ($L_N = 5\,\mathrm{mm}$) is the same for both (PC A and PC B) structures. The length of wider elements is different for both subsystems and represents correspondingly $L_N = 5\,\mathrm{mm}$ for PC A and $L_N = 15\,\mathrm{mm}$ for PC B. The structures (specimens) under study were prepared with the method of optical lithography.

The numerical calculation of the spatial field distribution at the frequency of Tamm state and the crowding of energy in the vicinity of interface can be seen in Figures 1(c), (d).

Numerical results and the corresponding experimental data for the transmission coefficient of electromagnetic waves through a composite line, modelling our system (see Figure 1(d)) are presented respectively in Figure 2 (correspondingly (a), (b) and (c)).

Experimental data: The transmission spectra obtained separately for two systems PC A with $N = 8$ (red solid line) and PC B with $N = 8$ (blue dashed line) (Figure 2(b)); Obtained experimentally transmission spectra for composite system (PC A + PC B) (Figure 2(c)). Overlapping forbidden bands for these structures are marked with shadowed area.

One can see from Figure 2 that band gaps for transmission spectra of each of structures PC A and PC B coincide in the vicinity of 10 GHz and 27 GHz. However, as can be seen from Figure 2(c), the composite structure (PC A+PC B) has two sharp transmittance peaks in the frequency regions near 10 and 30 GHz (overlapping forbidden bands for these structures are marked with shadowed area). Note that only one of them (the peak at 10 GHz) satisfies the matching conditions. It follows from Figure 2 that the experimental measurement of the transmission coefficient confirms the results of numerical

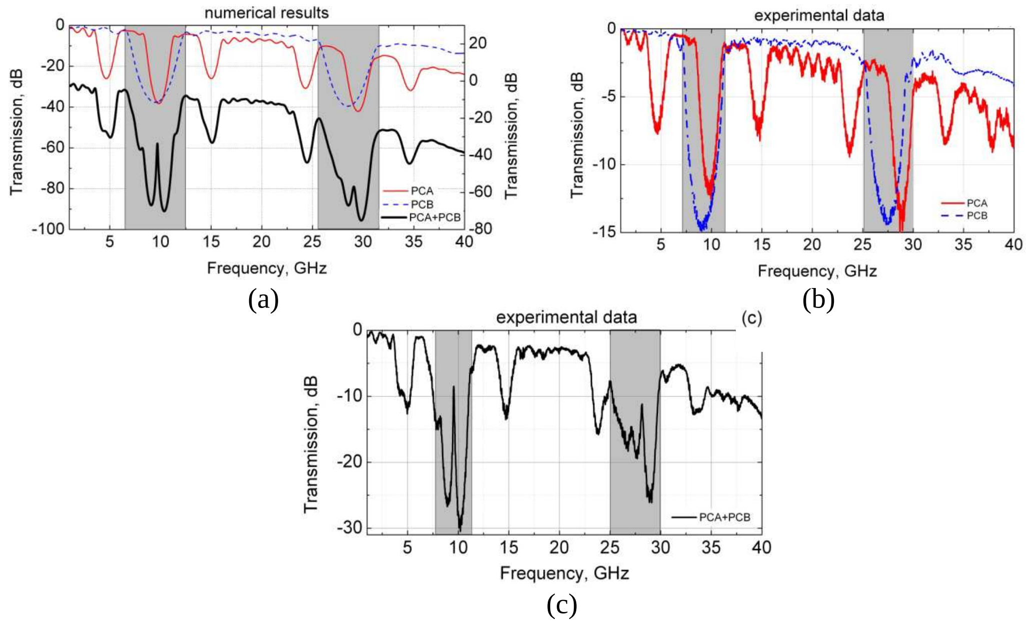

Figure 2. (Color online) The **numerical results** for transmission coefficients for the two sub-systems separately and for matched two-component system (Figure 2(a)). The calculation carried out for frequency domain (0–40 GHz). PC A — red line; PC B — blue dashed line (the axis is on the left side); PC A + PC B — black solid line (the axis is on the right side).

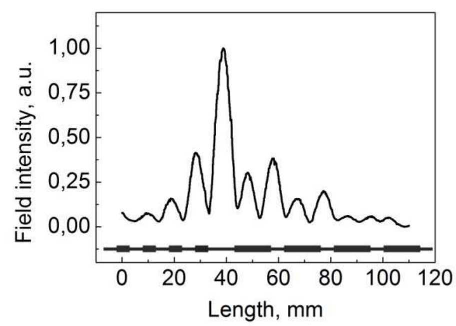

Figure 3. Crowding of the electromagnetic wave energy in the neighborhood of the boundary point separating two subsystems — experimental results.

calculations.

Parameters of forbidden bands in the zone overlapping area are correspondingly: for the zone PC A the boundaries are $f_1 = 8.2\,\mathrm{GHz}$, $f_2 = 11.3\,\mathrm{GHz}$, so the width of the forbidden band having the serial number 2 is $\Delta f_{\mathrm{PC\,A}} = 3.1\,\mathrm{GHz}$; the boundaries of the zone PC B are correspondingly $f_1 = 7.1\,\mathrm{GHz}$ $f_2 = 11.3\,\mathrm{GHz}$, so the width of the forbidden band having the serial number 1 is $\Delta f_{\mathrm{PC\,B}} = 4.2\,\mathrm{GHz}$.

A discrepancy in modes localization may be due to inaccuracy of structures manufacturing and neglecting of losses in the substrate. It may also be concerned with mathematical method used for calculation. Q of Tamm peaks: the calculated value is ($Q_C = 115$), and the experimental value is ($Q_E = 96$).

Figure 3 represents the experimental data for spatial energy distribution of electromagnetic field in the neighborhood of the boundary point separating two subsystems. A significant concentration of the field energy should be noted in the vicinity of this point.

During our experiment, the Vector Network Analyzer, which measures the transmission coefficient of microwave, and the scanner were connected to the computer. Specially developed software was controlling the scanning device: it moved test probe by 0.5 mm steps along the microstrip line and simultaneously (at every step) received the current value of microwave transmission coefficient (the transmission peak at the frequency located at the first band gap) obtained from the Vector Analyzer.

Making comparison of Figure 1(c) and Figure 3 we see that experimental data strongly support our numerical calculations and that the concentration of electromagnetic energy really takes place at the neighborhood of the boundary point separating two subsystems. We name this concentration of electromagnetic energy for the chain-like system as **an analogue of the Tamm state** (for details see [15]).

The concentration of the electromagnetic field near the boundary point of transition from one subsystem to another can have a number of important practical applications, and it allows, in particular, enhancement of effects accompanying the propagation of microwaves through a complex system (for details see [3]).

3. CONCLUSIONS

- the propagation of electromagnetic wave through a matched composite of periodical chain-like systems is studied numerically and experimentally. The composite system is modelled with two periodical chains of microstrip quadrupoles. The microwave experiments data strongly support the results of numerical calculations for the matched composite line consisting of two subsystems;
- it is shown that coincidence of forbidden bands of both subsystems results in a sharp narrow peak of transmission coefficient for electromagnetic waves. This peak is located in the forbidden frequency zone of matched composite system;
- observed experimentally concentration of electromagnetic field in the neighborhood of the boundary point separating two subsystems and likeness of geometry enables us to identify this peak as analogue of the Tamm state in chain-like system;

We should emphasize here that chain-like systems represent an interesting and important example of physically new periodical systems showing the analogue of Tamm states.

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