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STUDY OF SPECTRAL AND POLARIZATION CHARACTERISTICS OF LAYERED CHIRAL MEDIA

Study of layered chiral media is interesting both for fundamental and applied physics. Such media may be used for design of microwave devices such as filters and polarizers. Despite the fact that the spectral and polarization characteristics of the layered media are studied well enough now, the layered chiral media are studied insufficiently. In this paper we study the spectral and polarization characteristics of layered chiral media using the propagation matrix method. The coefficients of the propagation matrix of the periodically layered chiral medium are obtained. The transmission and reflection coefficients of linearly polarized electromagnetic waves for the structure consisting of planar chiral layers were calculated. The boundaries of the forbidden bands for a periodic medium, which unit cell consists of two different chiral layers were determined. It is shown that the boundaries of the forbidden bands do not depend on the chirality parameter of the layers of the structure. It was found that for certain values of the layers thicknesses, the forbidden bands width tends to zero. It is found that the proposed calculating method for the reflection and transmission coefficients can be used to determine the effective constitutive parameters of artificial chiral metamaterials. Fig.: 3. Ref.: 13 titles.

Key words: metamaterial, layered chiral structure, propagation matrix method.

At present, much attention is given to the study of electromagnetic waves propagation through the layered media [1–5]. But the study of properties of layered chiral media is still insufficient. However, the study of such media is interesting both for fundamental and applied physics. Layered chiral media can be used for design of magnetically controllable microwave devices such as filters, polarizers, etc. The study of layered chiral media, which include magnetically active elements is very important because the effective constitutive parameters of such media can be controlled by static magnetic field.

The aim of work is to study the electromagnetic waves propagation in layered chiral media using the propagation matrix method. The main attention is given to the determination of effective constitutive parameters of chiral metamaterials as analogs of optically active and magnetically active materials.

1. Calculation of spectral and polarization characteristics of layered chiral media using propagation matrix method.

To solve the problem we write the system of Maxwell's equations for harmonic electromagnetic fields in chiral media:

$$\begin{aligned} \operatorname{rot}\vec{E} &= -\frac{1}{c}\frac{\partial\vec{B}}{\partial t} = ik_0\vec{B}, \\ \operatorname{rot}\vec{H} &= \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{D}}{\partial t} = \frac{4\pi}{c}\vec{j} - ik_0\vec{D}, \end{aligned} \tag{1}$$

where $\vec{j}$ is the conduction current density vector, $\vec{E}$ and $\vec{H}$ are electric and magnetic field intensity vectors, $\vec{D}$ and $\vec{B}$ are electric and magnetic induction vectors, $k_0 = \omega/c$ is the propagation constant for a vacuum, ω is the angular frequency of the electromagnetic wave, c is the light velocity in vacuum.

We assume that the electric field intensity vector of a plane wave propagating along the z-axis has the following form:

$$\vec{E} = (E_x, E_y, 0)e^{i(kz-\omega t)}, \tag{2}$$

where k is the propagation constant for the medium.

To define the magnetic field vector $\vec{H}$ we find the vector $\vec{B}$ using expressions (1) and (2):

$$\vec{B} = (-E_y, E_x, 0)\frac{k}{k_0}e^{i(kz-\omega t)}. \tag{3}$$

We define the constitutive equations for a chiral medium as follows [6, 7]:

$$\begin{aligned} \vec{D} &= \varepsilon\vec{E} + i\kappa\vec{H}, \\ \vec{B} &= \mu\vec{H} - i\kappa\vec{E}, \end{aligned} \tag{4}$$

where ε and μ are dielectric permittivity and magnetic permeability, κ is the chirality parameter.

From equations (2)–(4), we find the vector $\vec{H}$:

$$\begin{aligned} \vec{H} = \frac{\vec{B} + i\kappa\vec{E}}{\mu} &= ((-E_y, E_x, 0)\frac{k}{k_0\mu} + \\ &+ i(E_x, E_y, 0)\frac{\kappa}{\mu})e^{i(kz-\omega t)}. \end{aligned} \tag{5}$$

Let us consider the case of normally propagation of plane electromagnetic waves through the layer of chiral medium with thickness d (Fig. 1).

In Fig. 1 layers 1 and 3 are represented by non-chiral medium with constitutive parameters ε_1, μ_1 and ε_3, μ_3. Layer 2 is represented by a chiral medium with constitutive parameters ε_2, μ_2 and κ.

Let the incident wave with the amplitude of E_1 be linearly polarized along the x axis. The expressions for the incident (index “inc”) and reflected (index “ref”) electromagnetic field in medium 1 are the following:

$$\begin{aligned}
&\vec{E}_1^{inc} = (E_1, 0, 0)e^{i(k_1 z - \omega t)},\\
&\vec{E}_1^{ref} = (E_{1x}^{ref}, E_{1y}^{ref}, 0)e^{i(-k_1 z - \omega t)},\\
&\vec{H}_1^{inc} = (0, E_1, 0)\, Z_1^{-1} e^{i(k_1 z - \omega t)},\\
&\vec{H}_1^{ref} = (E_{1y}^{ref}, -E_{1x}^{ref}, 0)\, Z_1^{-1} e^{i(-k_1 z - \omega t)},
\end{aligned} \qquad (6)$$

where $k_1 = k_0 n_1 = k_0\sqrt{\varepsilon_1\mu_1}$ and $Z_1 = \sqrt{\mu_1/\varepsilon_1}$ are the propagation constant and the characteristic impedance in medium 1.

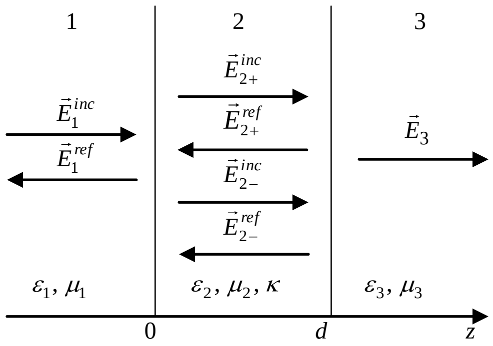

Fig. 1. The geometry of the problem

In chiral medium 2, there are two eigen waves with right (+) and left (–) circular polarization [7, 8]. The expressions for the electromagnetic waves in medium 2, propagating along the negative (index “ref”) and positive (index “inc”) direction of the axis z, are as follows:

$$\begin{aligned}
&\vec{E}_{2\pm}^{inc} = (1, \pm i, 0)E_{2x\pm}^{inc}\, e^{i(k_{2\pm} z - \omega t)},\\
&\vec{E}_{2\pm}^{ref} = (1, \pm i, 0)E_{2x\pm}^{ref} e^{i(-k_{2\mp} z - \omega t)},\\
&\vec{H}_{2\pm}^{inc} = (\mp i, 1, 0)\, Z_2^{-1} E_{2x\pm}^{inc}\, e^{i(k_{2\pm} z - \omega t)},\\
&\vec{H}_{2\pm}^{ref} = (\pm i, -1, 0)\, Z_2^{-1} E_{2x\pm}^{ref} e^{i(-k_{2\mp} z - \omega t)},
\end{aligned} \qquad (7)$$

where $k_{2\pm} = k_0 n_{2\pm} = k_0(\sqrt{\varepsilon_2\mu_2} \pm \kappa)$ and $Z_2 = \sqrt{\mu_2/\varepsilon_2}$ are the propagation constant and characteristic impedance for waves with right and left circular polarization in medium 2 [8].

In medium 3 the expression for the electromagnetic fields of the transmitted wave are the following:

$$\begin{aligned}
&\vec{E}_3 = (E_{3x}, E_{3y}, 0)e^{i(k_3 z - \omega t)},\\
&\vec{H}_3 = (-E_{3y}, E_{3x}, 0)\, Z_3^{-1} e^{i(k_3 z - \omega t)},
\end{aligned} \qquad (8)$$

where $k_3 = k_0 n_3 = k_0\sqrt{\varepsilon_3\mu_3}$ and $Z_3 = \sqrt{\mu_3/\varepsilon_3}$ are the propagation constant and characteristic impedance in medium 3.

Let us suppose that conditions for the constitutive parameters at which the electromagnetic field intensity in chiral medium 2 is limited at $z \to \infty$. For this purpose, considering that according to equations (7), the term $e^{ik_{2\pm} z}$ is a finite value at $z \to \infty$. Therefore, the constitutive parameters of the chiral layer must satisfy the following condition:

$$\mathrm{Im}\left(\sqrt{\varepsilon_2\mu_2}\right) \geq \left|\kappa''\right|. \qquad (9)$$

To find the unknown wave amplitudes E_{1x}^{ref}, E_{1y}^{ref}, E_{2x+}^{inc}, E_{2x-}^{inc}, E_{2x+}^{ref}, E_{2x-}^{ref}, E_{3x}, E_{3y} we use the boundary conditions of equality of the tangential component of the electric and magnetic field intensities at the chiral layer boundaries:

$$\begin{aligned}
&\{E_x(z = 0; d)\} = 0,\ \{E_y(z = 0; d)\} = 0,\\
&\{H_x(z = 0; d)\} = 0,\ \{H_y(z = 0; d)\} = 0.
\end{aligned} \qquad (10)$$

Let us write the amplitudes of the electric and magnetic field intensities at the boundary of layers 1 and 2. From boundary conditions (10) on the boundary $z = 0$ we obtain:

$$\begin{aligned}
&E_x(0) = E_{2x+}^{inc} + E_{2x-}^{inc} + E_{2x+}^{ref} + E_{2x-}^{ref},\\
&E_y(0) = i\,(E_{2x+}^{inc} - E_{2x-}^{inc} + E_{2x+}^{ref} - E_{2x-}^{ref}),\\
&H_x(0) = -i\, Z_2^{-1}(E_{2x+}^{inc} - E_{2x-}^{inc} - E_{2x+}^{ref} + E_{2x-}^{ref}),\\
&H_y(0) = Z_2^{-1}(E_{2x+}^{inc} + E_{2x-}^{inc} - E_{2x+}^{ref} - E_{2x-}^{ref}).
\end{aligned} \qquad (11)$$

By solving equations system (11) with respect to E_{2x+}^{inc}, E_{2x-}^{inc}, E_{2x+}^{ref} and E_{2x-}^{ref}, we obtain:

$$\begin{aligned}
&E_{2x\pm}^{inc} = \frac{E_x(0) \mp iE_y(0) \pm i\, Z_2 H_x(0) + Z_2 H_y(0)}{4},\\
&E_{2x\pm}^{ref} = \frac{E_x(0) \mp iE_y(0) \mp i\, Z_2 H_x(0) - Z_2 H_y(0)}{4}.
\end{aligned} \qquad (12)$$

Let us write the amplitudes of the electric and magnetic fields at the boundary of layers 2 and 3. From boundary conditions (10) on the boundary $z = d$ we obtain:

$$\begin{aligned}
&E_x(d) = E_{2x+}^{inc} e^{ik_{2+} d} + E_{2x-}^{inc} e^{ik_{2-} d} +\\
&+ E_{2x+}^{ref} e^{-ik_{2-} d} + E_{2x-}^{ref} e^{-ik_{2+} d},\\
&E_y(d) = i\,(E_{2x+}^{inc} e^{ik_{2+} d} - E_{2x-}^{inc} e^{ik_{2-} d} +\\
&+ E_{2x+}^{ref} e^{-ik_{2-} d} - E_{2x-}^{ref} e^{-ik_{2+} d}),\\
&H_x(d) = -i\, Z_2^{-1}(E_{2x+}^{inc} e^{ik_{2+} d} - E_{2x-}^{inc} e^{ik_{2-} d} -\\
&- E_{2x+}^{ref} e^{-ik_{2-} d} + E_{2x-}^{ref} e^{-ik_{2+} d}),\\
&H_y(d) = Z_2^{-1}(E_{2x+}^{inc} e^{ik_{2+} d} + E_{2x-}^{inc} e^{ik_{2-} d} -\\
&- E_{2x+}^{ref} e^{-ik_{2-} d} - E_{2x-}^{ref} e^{-ik_{2+} d}).
\end{aligned} \qquad (13)$$

By substituting $E_{2x\pm}^{inc}$ and $E_{2x\pm}^{ref}$ in expressions (13), we obtain:

$$E_x(d)=E_x(0)\cos(k_2d)\cos(k_0\kappa d)+$$
$$+E_y(0)\cos(k_2d)\sin(k_0\kappa d)-$$
$$-H_x(0)\,i\,Z_2\sin(k_2d)\sin(k_0\kappa d)+$$
$$+H_y(0)i\,Z_2\sin(k_2d)\cos(k_0\kappa d),$$
$$E_y(d)=-E_x(0)\cos(k_2d)\sin(k_0\kappa d)+$$
$$+E_y(0)\cos(k_2d)\cos(k_0\kappa d)-$$
$$-H_x(0)\,i\,Z_2\sin(k_2d)\cos(k_0\kappa d)-$$
$$-H_y(0)\,i\,Z_2\sin(k_2d)\sin(k_0\kappa d), \quad (14)$$
$$H_x(d)=E_x(0)\,i\,Z_2^{-1}\sin(k_2d)\sin(k_0\kappa d)-$$
$$-E_y(0)\,i\,Z_2^{-1}\sin(k_2d)\cos(k_0\kappa d)+$$
$$+H_x(0)\cos(k_2d)\cos(k_0\kappa d)+$$
$$+H_y(0)\cos(k_2d)\sin(k_0\kappa d),$$
$$H_x(d)=E_x(0)\,i\,Z_2^{-1}\sin(k_2d)\cos(k_0\kappa d)+$$
$$+E_y(0)\,i\,Z_2^{-1}\sin(k_2d)\sin(k_0\kappa d)-$$
$$-H_x(0)\cos(k_2d)\sin(k_0\kappa d)+$$
$$+H_y(0)\cos(k_2d)\cos(k_0\kappa d),$$

where $k_2=k_0n_2=k_0(\sqrt{\varepsilon_2\mu_2})$. The expression (14) can be written in matrix form as:

$$\begin{pmatrix}E_x(d)\\E_y(d)\\H_x(d)\\H_y(d)\end{pmatrix}=M\begin{pmatrix}E_x(0)\\E_y(0)\\H_x(0)\\H_y(0)\end{pmatrix}=$$
$$=\begin{pmatrix}M_{11}&M_{12}&M_{13}&M_{14}\\M_{21}&M_{22}&M_{23}&M_{24}\\M_{31}&M_{32}&M_{33}&M_{34}\\M_{41}&M_{42}&M_{43}&M_{44}\end{pmatrix}\begin{pmatrix}E_x(0)\\E_y(0)\\H_x(0)\\H_y(0)\end{pmatrix}, \quad (15)$$

where elements of matrix *M* are as follows:

$$M_{11}=\cos(k_2d)\cos(k_0\kappa d),$$
$$M_{12}=\cos(k_2d)\sin(k_0\kappa d),$$
$$M_{13}=-i\,Z_2\sin(k_2d)\sin(k_0\kappa d),$$
$$M_{14}=i\,Z_2\sin(k_2d)\cos(k_0\kappa d),$$
$$M_{21}=-M_{12},\ \ M_{22}=M_{11},$$
$$M_{23}=-M_{14},\ \ M_{24}=M_{13}, \quad (16)$$
$$M_{31}=-M_{13}Z_2^{-2},\ \ M_{32}=-M_{14}Z_2^{-2},$$
$$M_{33}=M_{11},\ \ M_{34}=M_{12},$$
$$M_{41}=M_{14}Z_2^{-2},\ \ M_{42}=-M_{13}Z_2^{-2},$$
$$M_{43}=-M_{12},\ \ M_{44}=M_{11}.$$

Thus, expression (15) relates the tangential components of the electromagnetic fields on the opposite boundaries of the chiral layer.

Let us introduce the column vector $\psi(z)=(E_x(z);\,E_y(z);\,H_x(z);\,H_y(z))^T$, composed from the tangential components of the electromagnetic field in the structure under study. Then its value on the boundary $z=0$ is the following:

$$\psi_1(0)=\begin{pmatrix}E_{x1}(0)\\E_{y1}(0)\\H_{x1}(0)\\H_{y1}(0)\end{pmatrix}=N\begin{pmatrix}E_1\\E_{1x}^{ref}\\E_{1y}^{ref}\\0\end{pmatrix}=$$
$$=\begin{pmatrix}E_1+E_{1x}^{ref}\\E_{1y}^{ref}\\Z_1^{-1}E_{1y}^{ref}\\Z_1^{-1}(E_1-E_{1x}^{ref})\end{pmatrix}, \quad (17)$$

where $N=\begin{pmatrix}1&1&0&0\\0&0&1&0\\0&0&Z_1^{-1}&0\\Z_1^{-1}&-Z_1^{-1}&0&0\end{pmatrix}$.

At the boundary $z=d$ we have:

$$\psi_3(d)=\begin{pmatrix}E_{x3}(d)\\E_{y3}(d)\\H_{x3}(d)\\H_{y3}(d)\end{pmatrix}=\begin{pmatrix}E_{3x}e^{ik_3d}\\E_{3y}e^{ik_3d}\\-Z_3^{-1}E_{3y}e^{ik_3d}\\Z_3^{-1}E_{3x}e^{ik_3d}\end{pmatrix}. \quad (18)$$

From expressions (17)–(18) we find the relation between the tangential components of the electric and magnetic field at the boundaries of the chiral layer:

$$\psi_2(d)=M\psi_2(0)=M\psi_1(0)=\psi_3(d)=$$
$$=M\,N\begin{pmatrix}E_1\\E_{1x}^{ref}\\E_{1y}^{ref}\\0\end{pmatrix}=\begin{pmatrix}E_{3x}e^{ik_3d}\\E_{3y}e^{ik_3d}\\-Z_3^{-1}E_{3y}e^{ik_3d}\\Z_3^{-1}E_{3x}e^{ik_3d}\end{pmatrix}. \quad (19)$$

From (19) we find the amplitudes of the electromagnetic field tangential components in medium 1 and 3:

$$E_{1x}^{ref}=-E_1\frac{(M_{11}+Z_1^{-1}M_{14})-Z_3(M_{41}+Z_1^{-1}M_{44})}{(M_{11}-Z_1^{-1}M_{14})-Z_3(M_{41}-Z_1^{-1}M_{44})},$$
$$E_{1y}^{ref}=0,$$
$$E_{3x}=(E_1(M_{11}+Z_1^{-1}M_{14})+ \quad (20)$$
$$+E_{1x}^{ref}(M_{11}-Z_1^{-1}M_{14}))e^{-ik_3d},$$
$$E_{3y}=(E_1(M_{21}+Z_1^{-1}M_{24})+$$
$$+E_{1x}^{ref}(M_{21}-Z_1^{-1}M_{24}))e^{-ik_3d}.$$

We find transmission and reflection coefficients of electromagnetic waves through the chiral layer using the energy flux density [9]:

$$\vec{S}=\frac{c}{8\pi}\mathrm{Re}[\vec{E}\times\vec{H}^*]. \quad (21)$$

Expressions for the transmission *T* and reflection *R* coefficient are as follows:

$$T = \frac{S_z^{tr}}{S_z^{inc}}, \quad R = \frac{S_z^{ref}}{S_z^{inc}}, \tag{22}$$

where S_z^{inc}, S_z^{ref}, S_z^{tr} are normal components of the energy flux density for the incident, reflected and transmitted waves correspondingly. Analytic expressions for these components are the following:

$$\begin{aligned} S_z^{tr} &= \frac{c}{8\pi}\mathrm{Re}(E_{3x}H_{3y}^{*} - E_{3y}H_{3x}^{*}) = \\ &= \frac{c}{8\pi Z_3}(|E_{3x}|^2 + |E_{3y}|^2), \\ S_z^{ref} &= \frac{c}{8\pi}\mathrm{Re}(E_{1x}^{ref}H_{1y}^{ref*} - E_{1y}^{ref}H_{1x}^{ref*}) = \\ &= \frac{c}{8\pi Z_1}\left|E_{1x}^{ref}\right|^2, \\ S_z^{inc} &= \frac{c}{8\pi Z_1}E_1^2. \end{aligned} \tag{23}$$

Note that as expected for non-absorbing chiral medium ($\varepsilon_2'' = 0$, $\mu_2'' = 0$, $\kappa'' = 0$) the relation $T + R = 1$ is satisfied. Besides, one more important conclusion should be made from these expressions: the transmission T and reflection R coefficients are independent of the chirality parameter.

Now, in accordance with the aim of our study let us find the rotation angle of the polarization plane θ for the linearly polarized electromagnetic wave as the ratio of the transmitted wave electromagnetic field tangential components in medium 3:

$$\theta = \mathrm{arctg}\left(\frac{E_{3y}}{E_{3x}}\right) = -\mathrm{arctg}(\mathrm{tg}(k_0\kappa' d)) = -k_0\kappa' d, \tag{24}$$

$$\pi/2 \le \theta \le \pi/2.$$

2. Study of the band structure of spectrum of electromagnetic waves propagated in the periodic layered chiral media. Note that expressions (20) are suitable for the determination of the transmitted and reflected waves of the electromagnetic field through the layered structure consisting of m chiral layers. Moreover, the propagation matrix M equals the product of the propagation matrices for single chiral layers, $(i = 1 \ldots m)$:

$$M = M_m M_{m-1} \ldots M_1, \tag{25}$$

where M_i are propagation matrices of single chiral layers with thicknesses d_i and constitutive parameters ε_i, μ_i, κ_i. Here, in expressions (20) value $d = d_m + d_{m-1} + \ldots + d_1$ is the total thickness of all layers. Matrix elements are calculated using formulas (16) with the following substitutions:

$$k_2 \to k_0(\sqrt{\varepsilon_i \mu_i}), \; Z_2 \to \sqrt{\mu_i / \varepsilon_i}, \; d \to d_i, \; \kappa \to \kappa_i.$$

The rotation angle of the polarization plane for the structure consisting of m chiral layers is defined as a sum of the rotation angles of the polarization plane for each chiral layer as:

$$\theta = -k_0(\kappa_1' d_1 + \kappa_2' d_2 + \ldots + \kappa_m' d_m). \tag{26}$$

Thus, it is possible to calculate the effective constitutive parameters and polarization characteristics of layered chiral structure using the transmission and reflection coefficients of electromagnetic waves and with help of the technique described in [10].

For the periodic structure/medium consisting of m unit cells, each of which contains two chiral layers, the resulting propagation matrix is as follows:

$$M_m = (M_2 M_1)^m, \tag{27}$$

where M_1 and M_2 are propagation matrixes for chiral layers of the unit cell.

We calculate the transmission (solid line) and reflection coefficients (dashed line) of electromagnetic waves for a periodic structure consisting of $m = 5$ unit cells (Fig. 2). Let us assume that $d_1 = 2.0$ mm, $d_2 = 2.0$ mm, $\varepsilon_1 = 5$, $\mu_1 = 1$, $\kappa_1 = 0.2$, $\varepsilon_2 = 2$, $\mu_2 = 1$, $\kappa_2 = 0$. As can be seen from Fig. 2, there are 7 specific areas, corresponding to forbidden bands for finite periodic structure under study on the dependence $T(\omega)$.

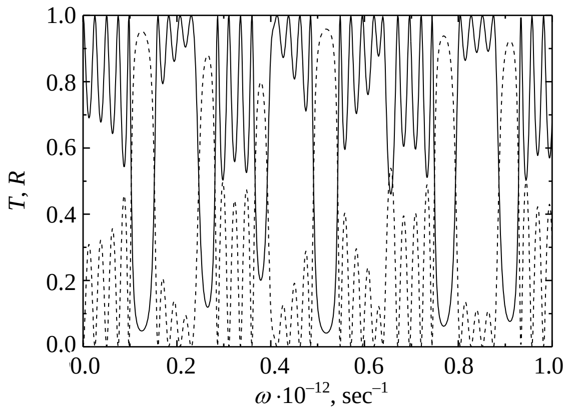

Fig. 2. Transmission and reflection coefficients of electromagnetic waves for chiral periodic structure with 5 elementary cells

In order to determine the boundaries of forbidden bands in the transmission coefficient spectrum, it is necessary to solve the equation that relates the tangential components of the amplitudes of the electric and magnetic fields on the boundaries of the unit cell of the infinite periodic structure (Floquet's theorem [3, 11]):

$$M\psi = \lambda\psi, \tag{28}$$

where $M = M_2 M_1$ is the propagation matrix of the periodic structure unit cell, ψ is the vector consisting of the tangential components of the electric and magnetic field, λ is the eigenvalue of matrix M.

Transforming (28), we obtain:

$$(M - I\lambda)\psi = 0, \tag{29}$$

where I is the unit matrix.

Calculating the determinant $\left|M - I\lambda\right| = 0$ and grouping the factors relative to λ, we obtain an equation of the fourth degree:

$$\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0, \quad (30)$$

where

$$a_0 = \left|M\right| = 1, \;\; a_1 = a_3 = -2(M_{11} + M_{33}),$$

$$a_2 = M_{11}^2 + M_{12}^2 + M_{33}^2 + M_{34}^2 - - 2(M_{13}M_{31} - M_{14}M_{32}) + 4M_{11}M_{33}.$$

Because of $a_1 = a_3$, equation (30) can be written as a product of two quadratic equations:

$$(\lambda^2 + q_1\lambda + 1)(\lambda^2 + q_2\lambda + 1) = \lambda^4 + (q_1 + q_2)\lambda^3 + + (q_1q_2 + 2)\lambda^2 + (q_1 + q_2)\lambda + 1 = 0, \quad (31)$$

From (31) we find the relation between parameters q_1, q_2 with parameters a_1, a_2 [11, 12] as:

$$q_{1,2} = \frac{a_1}{2} \pm \sqrt{\left(\frac{a_1}{2}\right)^2 + 2 - a_2}\,. \quad (32)$$

By solving equation (31), we obtain two pairs of roots:

$$\lambda_{1,2} = -\frac{q_1}{2} \pm \sqrt{\left(\frac{q_1}{2}\right)^2 - 1},$$
$$\lambda_{3,4} = -\frac{q_2}{2} \pm \sqrt{\left(\frac{q_2}{2}\right)^2 - 1}. \quad (33)$$

We group the solutions of equation (31) so that $\lambda_1 = \lambda_2^* = e^{ik_{b1}d}$ and $\lambda_3 = \lambda_4^* = e^{ik_{b2}d}$, where $d = d_1 + d_2$ is the thickness of the unit cell. Then we have the following relations:

$$\cos(k_{b1,2}d) = -\frac{q_{1,2}}{2}, \quad (34)$$

where k_{b1} and k_{b2} are Bloch wave numbers [3].

Bloch wave numbers are real in the propagation band and complex numbers in the forbidden band. Let us rewrite expression (34) as the following:

$$\cos(k_{b1,2}d) = \cos((\arccos D) \mp \theta), \quad (35)$$

where

$$D = \cos(k_0n_1d_1)\cos(k_0n_2d_2) - ((Z_1^2 + Z_2^2)/(2Z_1Z_2))\sin(k_0n_1d_1)\sin(k_0n_2d_2),$$
$$\theta = -k_0(d_1\kappa_1 + d_2\kappa_2).$$

The expression for D determines the Bloch wave number for the non-chiral layered periodic structures with the same values of $n_1 = \sqrt{\varepsilon_1\mu_1}$, $n_2 = \sqrt{\varepsilon_2\mu_2}$, $Z_1 = \sqrt{\mu_1/\varepsilon_1}$, $Z_2 = \sqrt{\mu_2/\varepsilon_2}$. The expression for θ determines the rotation angle of the polarization plane for the unit cell of layered periodic chiral structure. When $\left|D\right| > 1$, the value of $\arccos D$ becomes complex, which corresponds to the band gap. When $\left|D\right| \le 1$, the value of $\arccos D$ is real. At the same frequencies we have the allowed band. Thus, the boundaries of the forbidden bands are defined by the condition $\left|D\right| = 1$. Note that this condition does not depend on the chirality parameter of the layers in the structure under study.

We study the dependence of the boundaries of the forbidden bands for chiral periodic structure as function on d_1/d ration for fixed $d = 4.0$ mm (Fig. 3). These boundaries are shown in Fig. 3, and their positions are determined from the condition $\left|D\right| = 1$. The dashed areas correspond to the forbidden bands of the chiral periodic structure.

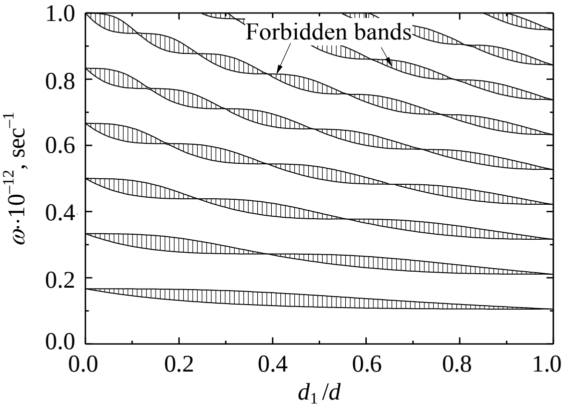

Fig. 3. Dependence of the forbidden bands areas on the parameter d_1/d for the chiral periodic structure for a fixed value of $d = 4.0$ mm

In Fig. 3 we show that at certain relations between the parameters of the chiral layered periodic structure, the width of the forbidden bands becomes 0 [13]. This occurs when the thickness d_1 and d_2 is an integer number of half wavelengths. In this case we have the following conditions:

$$\sin(k_0n_1d_1) = 0,$$
$$\sin(k_0n_2d_2) = 0. \quad (36)$$

These conditions imply that:

$$k_0n_1d_1 = m_1\pi,$$
$$k_0n_2d_2 = m_2\pi, \quad (37)$$

where $m_1, m_2 = 1, 2, 3, \ldots$ are positive integers. Excluding the frequency from equations (37), we find the following relation for thicknesses d_1 and d_2:

$$\frac{d_1}{d_2} = \frac{m_1}{m_2}\frac{n_2}{n_1}. \quad (38)$$

The frequencies at which the forbidden bands have zero widths are defined as follows:

$$\omega_{m_1,m_2} = \frac{c\pi(m_1n_2 + m_2n_1)}{d\,n_1n_2}. \quad (39)$$

Thus, choosing the size and the refractive indexes of the layers of the periodic chiral structure unit cell, we can control the frequencies with zero width of band gaps.

Conclusions. The processes of the electromagnetic waves propagation in the layered chiral media using the propagation matrix method were studied. The coefficients of the propagation matrix for the periodically layered chiral medium were obtained. The transmission and reflection coefficients of the electromagnetic waves in the structure consisting of a finite number of planar chiral layers were calculated.

The boundaries of the forbidden bands of the periodic chiral medium whose unit cell consists of two planar chiral layers were determined. It is shown that these boundaries do not depend on the chirality parameter of the layers within the structure. It was found that for the certain values of chiral layers thicknesses the width of band gap becomes 0.

The studies presented above allow us to calculate the effective constitutive parameters of layered chiral media in order to design the artificial metamaterials with predetermined features.

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ИССЛЕДОВАНИЕ СПЕКТРАЛЬНЫХ И ПОЛЯРИЗАЦИОННЫХ ХАРАКТЕРИСТИК СЛОИСТЫХ КИРАЛЬНЫХ СРЕД

Исследование слоистых киральных сред представляет интерес и для фундаментальной и для прикладной физики. Такие среды могут применяться для создания СВЧ-устройств, таких как поляризаторы и фильтры. Несмотря на то что спектральные и поляризационные характеристики слоистых сред в настоящее время изучены достаточно хорошо, слоистые киральные среды изучены еще недостаточно. Нами исследованы спектральные и поляризационные характеристики слоистых киральных сред с использованием метода матриц распространения. Получены коэффициенты матрицы распространения для слоисто-периодической киральной среды. Вычислены коэффициенты прохождения и отражения линейно поляризованных электромагнитных волн для структуры, состоящей из плоских киральных слоев. Определены границы запрещенных зон для периодической среды, элементарная ячейка которой состоит из двух различных киральных слоев. Показано, что границы запрещенных зон не зависят от параметра киральности слоев, входящих в структуру. Найдено, что при определенных значениях толщин слоев ширина запрещенных зон становится равной 0. Отмечено, что предложенную методику расчета коэффициентов отражения и прохождения можно использовать для определения эффективных материальных параметров искусственных киральных метаматериалов.

Ключевые слова: метаматериал, слоистая киральная структура, метод матриц распространения.

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ДОСЛІДЖЕННЯ СПЕКТРАЛЬНИХ І ПОЛЯРИЗАЦІЙНИХ ХАРАКТЕРИСТИК ШАРУВАТИХ КІРАЛЬНИХ СЕРЕДОВИЩ

Дослідження шаруватих кіральних середовищ цікаве для фундаментальної і прикладної фізики. Такі середовища можуть застосовуватися для створення НВЧ-пристроїв, таких як поляризатори і фільтри. Незважаючи на те що спектральні й поляризаційні характеристики шаруватих середовищ на сьогодні вивчені досить добре, шаруваті кіральні середовища вивчені ще недостатньо. Нами досліджено спектральні й поляризаційні характеристики шаруватих кіральних середовищ з використанням методу матриць поширення. Отримано коефіцієнти матриці поширення для шарувато-періодичного кірального середовища. Обчислено коефіцієнти проходження та відбиття лінійно поляризованих електромагнітних хвиль для структури, що складається з плоских кіральних шарів. Визначено межі заборонених зон для періодичного середовища, елементарна комірка якої складається з двох різних кіральних шарів. Показано, що межі заборонених зон не залежать від параметра кіральності шарів, що входять в структуру. Знайдено, що при певних значеннях товщин шарів ширина заборонених зон стає рівною 0. Відзначено, що запропоновану методику розрахунку коефіцієнтів відбиття і проходження можна використовувати для визначення ефективних матеріальних параметрів штучних кіральних метаматеріалів.

Ключові слова: метаматеріал, шарувата кіральна структура, метод матриць поширення.