Transmission of electromagnetic waves in a magnetic fine-stratified structure

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The specific features of the transmission spectra of the TE-wave normally incident from the uniform medium onto the fine-stratified structure fabricated by periodic alternating ferrite and semiconductor layers were investigated. The expressions for effective permeability and permittivity were obtained in the long-wave limit. It was shown that, in the investigated multilayer composite, the left-handed behavior is possible. The dependence of transmission spectra of electromagnetic waves that propagate through a fine-stratified magnetoactive structure on the external magnetic field was examined experimentally. © 2009 Optical Society of America

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1. INTRODUCTION

The propagation of electromagnetic waves in left-handed media (LHM) has been thoroughly investigated by many researchers [1–3], and various new phenomena have been discovered. Left-handed materials do not exist naturally; however, it is possible to make artificial metamaterials with negative ϵ and μ in the same frequency range [4–6]. It was shown that, in periodic ferrite-semiconductor multilayer composite, the left-handed behavior is possible [7–9].

In this paper, we present the theoretical and experimental study of the peculiarities of transmission spectra of electromagnetic waves that propagate through a fine-stratified ferrite-semiconductor structure. The dependencies of effective permittivity and permeability of multilayer media on the frequency, parameters of the layers, and an external magnetic field are discussed. The influence of the dissipative processes on the properties of the reflection and transmission is considered.

2. STATEMENT OF THE PROBLEM AND BASIC EQUATIONS

Let us analyze the transmission of the plane-wave through the magnetoactive fine-layered periodic structure. Let us consider an infinite periodic structure where ferrite layers of thickness d_1 and semiconductor layers of thickness d_2 alternate (Fig. 1). Let the structure be placed into a magnetic field $\mathbf{H}_0$ parallel to the y axis. The z axis runs perpendicularly to the boundaries of layers. Assume that the thickness of the structure is L ($L=Nd$, where N is the number of periods, $d=d_1+d_2$ is the period of the structure). Assume the structure to be placed between homogeneous media with the dielectric permittivities ϵ_a and ϵ_b.

Electromagnetic processes in this structure are described by the Maxwell equations and by the equations of continuity. We seek the solutions in these equations in the form of $\exp(ik_x x+ik_{z1,z2}z-i\omega t)$, where $k_{z1}=\sqrt{(\omega^2\mu_F\epsilon_f)/c^2-k_x^2}$ and $k_{z2}=\sqrt{(\omega^2\epsilon_{yy})/c^2-k_x^2}$ are transversal wave numbers; ϵ_f is the permittivity of the ferrite layer; $\epsilon_{yy}=\epsilon_0[1-\omega_p^2/(\omega(\omega+i\nu))]$ is the permittivity of the semiconductor layer, ϵ_0 is the part of the permittivity attributed to the lattice, ω_p is the plasma frequency, ν is the collision frequency, $\mu_F=\mu_\parallel+(\mu_\perp^2/\mu_\parallel)$ is the effective permeability of the ferrite layer. The permeability of the ferrite layer is a tensor characteristic for the investigated microwave region. It can be written as [10]

$$\mu_\parallel=\mu_{xx}=\mu_{zz}=1+\frac{\omega_M(\omega_H^2+\omega_r^2-i\omega_r\omega)}{\omega_H(\omega_H^2+\omega_r^2-\omega^2-2i\omega_r\omega)},$$

$$\mu_\perp=\mu_{xz}=-\mu_{zx}=-\frac{i\omega\omega_M}{\omega_H^2+\omega_r^2-\omega^2-2i\omega_r\omega},$$

where $\omega_M=2\pi egM/(mc)$, $\omega_H=egH_0/(2mc)$, g is the factor of spectroscopic splitting, M is the saturation magnetization, and ω_r is the relaxation frequency. The tensor of permeability for the nonmagnetic semiconductor is $\mu_{ii}=1$.

We will assume that the structure is homogeneous in the x and y directions and put $\partial/\partial y=0$, omitting the dependence on the coordinate y in the equations. Then, Maxwell's equations split into independent equations for two modes with different polarizations. We consider the TE-polarization with components H_x,H_z,E_y. Using the method of the transmission matrix (which relates the fields at the beginning of a wave period and at its end) and applying the Floquet theorem, which takes into account the periodicity of the structure, we obtain the following dispersion relation for the infinite periodic structure:

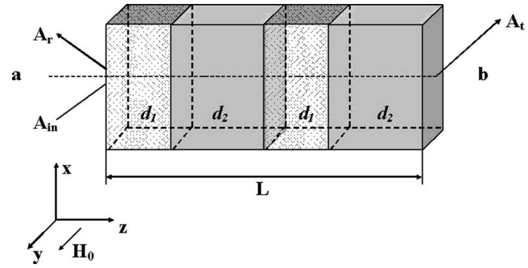

Fig. 1. Geometry of the problem.

$$\cos \bar{k}d = \cos k_{z1}d_1 \cos k_{z2}d_2 - \frac{1}{2}\left[\frac{k_{z1}}{k_{z2}}\frac{1}{\mu_F} + \frac{k_{z2}}{k_{z1}}\mu_F - \frac{k_x^2}{k_{z1}k_{z2}\mu_F}\left(\frac{\mu_\perp}{\mu_\parallel}\right)^2\right] \times \sin k_{z1}d_1 \sin k_{z2}d_2. \quad (1)$$

Here $\bar{k}$ is the averaged wave number describing the periodicity of the structure (Bloch wave number); $d=d_1+d_2$ is the period of the structure. The analysis of expression for effective permeability of ferrite layer μ_F reveals two characteristic frequencies: $\omega_1=\sqrt{\omega_H(\omega_H+\omega_M)}$ is the frequency of ferromagnetic resonance ($\mu_F \to 0$); $\omega_2=\omega_H+\omega_M$ is the frequency of antiresonance ($\mu_F=0$). From the dispersion equation (1) we obtain a third characteristic frequency: $\omega_3=\sqrt{\omega_H^2+\omega_M^2/2+\omega_H\omega_M}$. At this frequency the denominator of Eq. (1) is equal to zero. The frequencies ω_1 and ω_3 determine the asymptotes of the dispersion curves.

We study Eq. (1) in the case of a fine-layered medium; i.e., we assume that $k_{z1}d_1, k_{z2}d_2 \ll 1$. As the result we can expand the trigonometrical functions into the series. The Bloch wave number $\bar{k}=k_z$ is the transverse wave vector of a bigyrotropic medium [11], while the dispersion relation for a fine-layered semiconductor structure is written as

$$k_z^2 \frac{\mu_{zz}^*}{(\mu_{xx}+\alpha\mu_{zz}^*)} + k_x^2 = \frac{\omega^2}{c^2}\mu_{zz}\epsilon^*, \quad (2)$$

where $\mu_{xx}=(d_1\mu_F+d_2)/d$, $\mu_{zz}=\mu_{xx}\mu_{zz}^*/(\mu_{xx}+\alpha\mu_{zz}^*)$, $\mu_{zz}^*=d\mu_F/(d_2\mu_F+d_1)$, $\alpha=(d/\mu_F d_1 d_2)(\mu_\perp/\mu_\parallel)^2$, $\epsilon^*=(d_1\epsilon_f+d_2\epsilon_{yy})/d$. The electrodynamics properties of the structure in an anisotropic medium depend heavily on the direction of the wave propagation. The presence of a constant magnetic field brings about a number of specific features in the interaction of waves. If $k_x=0$, the dispersion relation takes the form [9]

$$k_z^2 = \frac{\omega^2}{c^2}\mu^*\epsilon^*, \quad (3)$$

where $\mu^*=\mu_{xx}$. Figure 2 shows frequency dependencies of the effective parameters $\epsilon^*(f)$ (dashed curve) and $\mu^*(f)$ (solid curve); here $f=\omega/2\pi$. In the calculations, we used the following numerical values of the parameters: $\epsilon_f=11.1$, $\omega_M=84.49$ GHz, $d_2=0.5$ mm, $\epsilon_0=17.8$, $\omega_p=100$ GHz, $g=2$, $d_1=0.5$ mm, $H_0=5000$ Oe. We ignore the collision frequency in the semiconductor layers and magnetic damping in the ferrite. In Fig. 1(a), in the frequency range $\omega_1<\omega<\omega_4$ ($\omega_4=\sqrt{(\omega_H+\omega_M d_1/d)(\omega_H+\omega_M)}$), the effective permeability μ^* is negative. For $\omega<\omega_5$ ($\omega_5=\omega_p\sqrt{\epsilon_0 d_2}/\sqrt{\epsilon_0 d_2+\epsilon_f d_1}$) $\epsilon^*<0$. Hence, for $\omega_4<\omega_5$, in the frequency range $\omega_1<\omega<\omega_4$ we have the composite with the left-handed behavior. Let us examine the effect of dissipation on the effective permittivity and permeability of the fine-layered structure. Fig. 2(b) displays dependencies $\epsilon^*(f)$ and $\mu^*(f)$ at $\omega_r=0.106$ GHz, $\nu=10$ GHz.

To control the left-handed properties of the structure, we should change the external magnetic field. Let us consider the possibility of the ideal LHM production. It can be interesting, because such structure creates the ideal image. It is known that the effective parameters of an ideal LHM are $\epsilon^*=\mu^*=-1$. It was obtained that $\epsilon^*=-1$ at frequency $\omega_{id}=\omega_p\sqrt{\epsilon_0 d_2}/\sqrt{\epsilon_0 d_2+\epsilon_f d_1-d}$. Figure 3 shows the dependence of the effective permeability on the external magnetic field. The calculation was made for the structure where $\epsilon_f=5.5$, $\omega_M=31.1$ GHz, $\epsilon_0=17.8$, ω_p

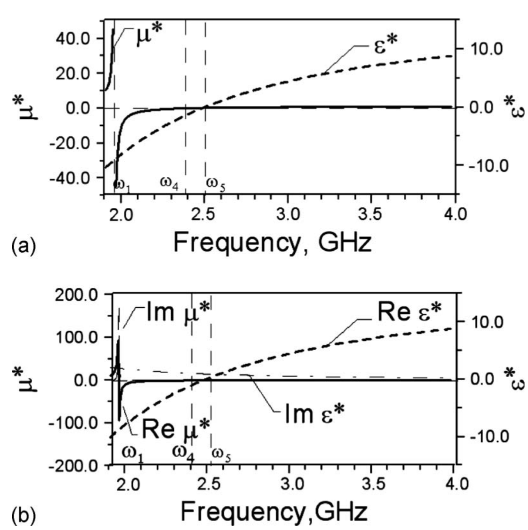

Fig. 2. The frequency dependencies of the effective parameters.

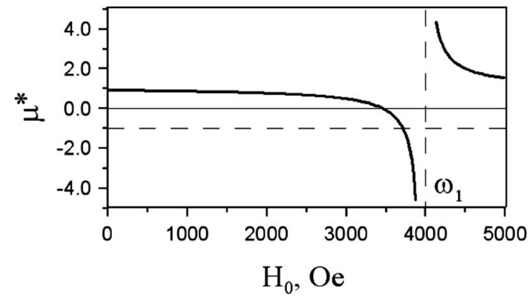

Fig. 3. The effective permeability as a function of external magnetic field.

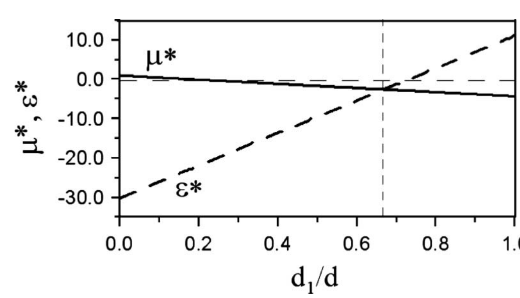

Fig. 4. The effective permeability and permittivity as a function of the thickness ratio.

$=100$ GHz, $g=2$, $d_1=d_2=0.0125$ mm, $f=13.3$ GHz. It can be seen that, for magnetic field $H_0 \approx 3700$ Oe, we have the ideal LHM. The effective parameters versus relative thickness of the first of the alternating layers are depicted in Fig. 4. In the calculations, we used the following numerical parameters values: $\epsilon_f=11.1$, $\omega_M=84.49$ GHz, $\epsilon_0=17.8$, $H_0=4340$ Oe, $\omega_p=100$ GHz, $g=2$, and $f=19.5$ GHz.

As the numerical calculations show, in the thickness diapason $0.19 < d_1/d < 0.73$ we obtain the left-handed structure.

3. TRANSMISSION SPECTRA OF THE STRUCTURE

Let us assume that the structure is placed between homogeneous media with the dielectric permittivities ϵ_a and ϵ_b. Consider the case of normal incidence of plane electromagnetic wave ($k_x=0.$). Assume that $k_{za,zb}=(\omega/c)\sqrt{\epsilon_{a,b}}$. Using the boundary conditions for tangential components of the electromagnetic field at $z=0$ and $z=Nd$, we arrive at the expressions for reflection and transmission coefficients:

$$R = \frac{k_z\mu^* \cos k_zL(k_{za}-k_{zb}) + i \sin k_zL(k_z^{\,2} - k_{za}k_{zb}\mu^{*2})}{k_z\mu^* \cos k_zL(k_{za}+k_{zb}) - i \sin k_zL(k_z^{\,2} + k_{za}k_{zb}\mu^{*2})},$$

$$T = \frac{2e^{-ik_{zb}L}k_zk_{za}\mu^*}{k_z\mu^* \cos k_zL(k_{za}+k_{zb}) - i \sin k_zL(k_z^{\,2} + k_{za}k_{zb}\mu^{*2})}. \tag{4}$$

Let us determine the conditions under which the reflectance is equal to zero. Assume that the periodic structure is placed into the vacuum $\epsilon_a=\epsilon_b=1$. In this case $|R|^2=0$ ($|T|^2=1$) if $Nk_zd=\pi q$, $q=0,\pm 1,\pm 2 \ldots$. It implies that the Wolf–Bragg resonance takes place; under this condition the thickness of the structure is equal to an integer number of half-waves. It is obvious that number of the Wolf–Bragg resonances depends on the thickness of the structure. From the formulas (4), it follows that if the condition $\epsilon_a=\epsilon^*/\mu^*$ takes place, we obtain the additional point of full transmittance. Let us call this point the impedance equality peak (IEP).

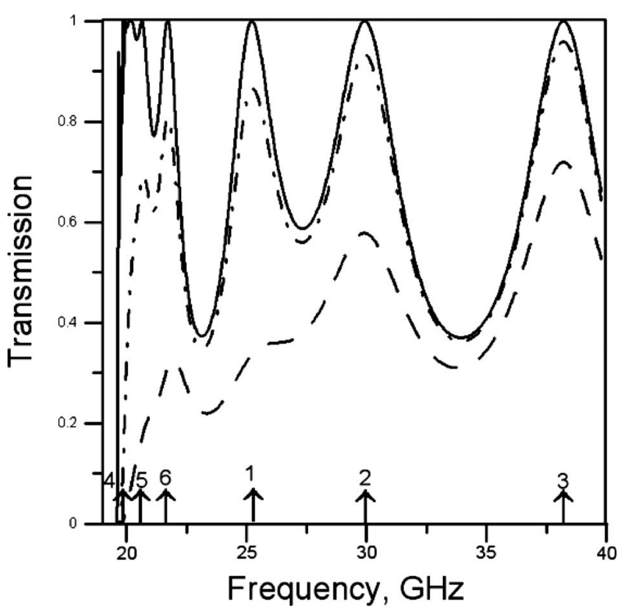

Fig. 5. $|T|^2$ as a function of the frequency f for various collision frequencies.

Figure 5 shows the transmission $|T|^2$ versus frequency at $H_0=5000$ Oe. When constructing Fig. 5, we took $\epsilon_f=11.1$, $\omega_M=84.49$ GHz, $\epsilon_0=17.8$, $\omega_p=100$ GHz, $g=2$, $d_1=0.5$ mm, and $d_2=0.5$ mm. The calculations were performed for three periods ($N=3$), $\epsilon_a=\epsilon_b=1$ (i.e., the uniform media are vacuum). The solid curve is given for $\omega_r=0$, $\nu=0$; chain curve corresponds to $\omega_r=0.106$ GHz, $\nu=1$ GHz. The dashed curve is for $\omega_r=0.106$ GHz, $\nu=10$ GHz. It can be seen that when the dissipation is taken into account, the transmission decreases with increasing collision frequency in the semiconductor layer. The frequency dependencies of the effective ϵ^* and μ^* are presented in Fig. 2. The frequency range $24.9\text{ GHz} < f < 40$ GHz corresponds to the right-handed material (RHM). First maximum (peak number 1 in Fig. 5) of the transmission at frequency $f \approx 25.2$ GHz (solid curve) is connected with an implementation of the condition for the IEP in the RHM. The rest of the maxima (2 and 3) are explained by the Wolf-Bragg

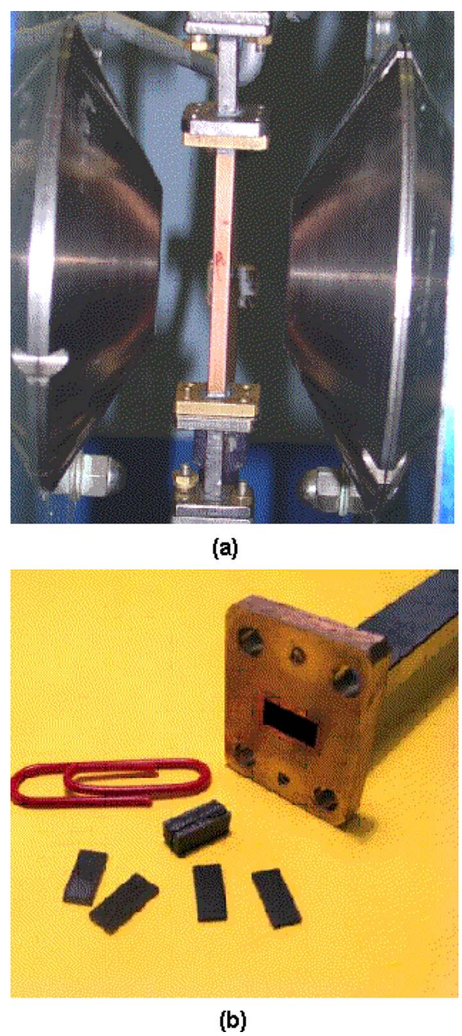

Fig. 6. (Color online) The investigated composite structure between (a) the magnet poles and (b) its components with the waveguide.

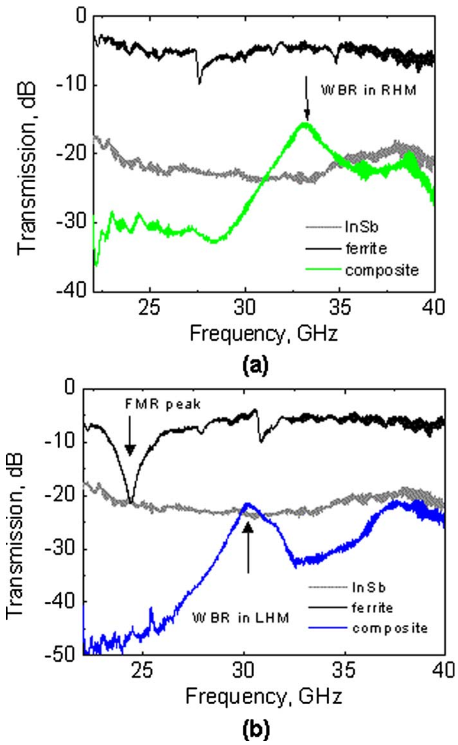

Fig. 7. (Color online) Transmission spectra of various structures: (a) $H_0=0$ Oe: Wolf-Bragg resonance (WBR) in RHM; (b) $H_0=6570$ Oe: WBR in LHM (1—ferrite layer; 2—InSb layer; 3—ferrite/semiconductor composite).

resonances (WBR in RHM). At the range $19.6\text{ GHz}<f<23.8\text{ GHz}$, we obtain the left-handed material. It can be seen that in this diapason we have some additional peaks of transmission. The maximum 4 of the transmission (solid curve) is connected with an implementation of the condition $\epsilon^*/\mu^*=1$ for IEP in LHM. The peaks 5 and 6 are explained by the Wolf–Bragg resonances in the LHM (WBR in LHM).

4. EXPERIMENT AND ANALYSIS

The one-dimensional (1D) periodic structure was used for experiment (Fig. 6). The composite structure is formed by three periods. The geometrical dimensions of the layers had the constitutive parameters of ferrite (brand 1SCH4)

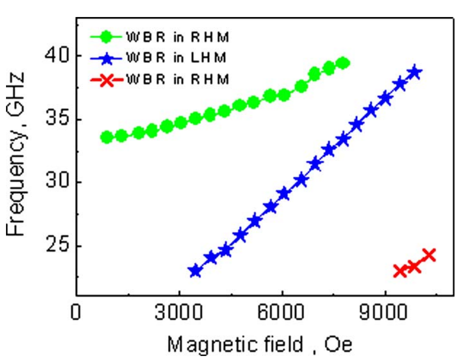

Fig. 8. (Color online) Frequency dependence of experimental resonance peaks position on magnetic field. One can watch as well an appearance of low-frequency mode (satellite one) of Wolf-Bragg resonance (WBR) in RHM at $H>9000$ Oe (red crosses).

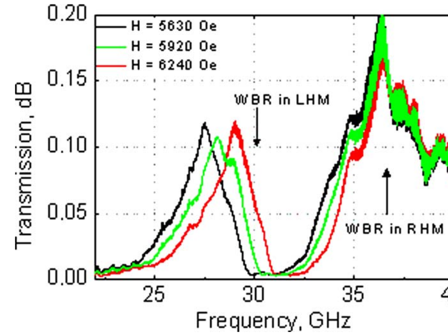

Fig. 9. (Color online) Transmission spectra of ferrite/semiconductor composite structure at various fields.

and InSb semiconductor corresponding to the modelled situation, described in Fig. 5. The composite structure was embedded into hollow rectangular metal waveguide with cross-section $7.2\times3.4\text{ mm}^2$.

The transmission coefficient was measured in the 22–40 GHz frequency range. The external static magnetic field $\mathbf{H}_0$ was changed in the range of 0–11000 Oe and applied normally to alternating magnetic field (Fig. 1). The experimental technique is presented in details in [12].

In Fig. 7 the results of experimental investigations of the transmission spectra of composite structure and its components are presented. Lower curves correspond to the spectra of composites, top curves describe the spectra of InSb plate (thickness of 0.5 mm) and middle ones are for the spectra of the ferrite plate (thickness of 0.5 mm). Note the expected ferromagnetic resonance (FMR) peak appears in spectra for ferrite specimen in Fig. 7(b). One can see that for magnetic field $H_0=0$ Oe the same transmission peak appears in the spectrum of the composite structure [Fig. 7(a)]. According to the calculations above, the origin of this peak should be associated with WBR in RHM. For large magnetic fields (for example, $H_0=6570$ Oe) the transparency resonance peak, which should be associated with WBR in LHM, appears [Fig. 7(b)].

The detuning of WBR in RHM and WBR in LHM with the magnetic field is shown in Fig. 8. It should be noted that the steepness of the curve of detuning WBR in RHM ($\partial f_{\mathrm{WBRinRHM}}/\partial H=1{,}02$ GHz/kOe) by the magnetic field is less than the corresponding steepness of detuning WBR in LHM ($\partial f_{\mathrm{WBRinRHM}}/\partial H=2{,}48$ GHz/kOe). Such behavior can be explained by the strong dependence of WBR in LHM position on the effective negative permeability region associated with ferromagnetic resonance behavior.

This scenario is shown in Fig. 9, where the variation of the spectra for the structure studied is presented as a function of the magnetic field. One can see easily that WBR in LHM peak shift is more noticeable than that of WBR in RHM. Let us note that the experimental peak marked as WBR in LHM coincides with peak 6 in Fig. 5, and the peak marked as WBR in RHM coincides with peak 1 in Fig. 5 with accuracy of the experiment. Thus we can conclude that the theoretical model developed in the paper rather well describes the main electromagnetic processes in the structure under study. It is necessary to note as well that we carried out experiments at room temperature; the IEP peaks didn't manifest themselves in the spectra detected. The obvious reason for this is a rather high damping in the layers.

5. CONCLUSIONS

In summary, we have presented the theoretical and experimental investigation of electromagnetic wave transmission and reflection by a fine–stratified periodic ferrite-semiconductor structure.

(1) Wolf–Bragg resonances have been found, separated, and identified for LHM and RHM.

(2) It has been shown that the electromagnetic field transmission through the magnetoactive structure can be controlled by the external magnetic field.

(3) It has been demonstrated that resonance processes in LHM depend more strongly on the magnetic field than ones in RHM.

(4) The effect of dissipation in the layers on the transmission of electromagnetic waves has been examined.

(5) Good coincidence between theory and experiment has been demonstrated.

The presence of resonance phenomena of different origin in the studied structure allows their use for numerous practical and technology applications, for example, as electronically controllable microwave and optical devices, etc.

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