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Tamm states in magnetophotonic crystals and permittivity of the wire medium

D P Belozorov[1], **M K Khodzitsky**[2] **and S I Tarapov**[2]

[1] Institute for Theoretical Physics NSC, Kharkov Institute of Physics & Technology, NAS of Ukraine, 1 Akademicheskaja St., Kharkov, 61108, Ukraine
[2] Institute of Radiophysics and Electronics, NAS of Ukraine, 12 Ac. Proskura St., Kharkov, 61085, Ukraine

E-mail: khodzitskiy@ire.kharkov.ua

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Abstract
The system consisting of a magnetophotonic crystal (MPC) and a medium with negative permittivity was studied both experimentally and theoretically. The medium was prepared as a set of conductive wires—wire medium (WM). The MPC elementary cell comprises three layers (air, ferrite and quartz). The Tamm state (TS) position in a stop band was calculated depending on the thickness of the air layer in an MPC elementary cell and on the value of the WM negative permittivity. Using the experimental value of the TS frequency the inverse problem was solved and the effective permittivity of the WM bounding the MPC was obtained.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Recently, so-called Tamm states (TSs) have attracted considerable attention of researchers studying electromagnetic processes in magnetophotonic crystals (MPC) [1–4]. The possibility of a TS in MPCs on the interface with a negative permittivity medium (NPM) was predicted theoretically in [3] and observed experimentally for optical frequencies in [4]. A characteristic feature of these states is the concentration of the electromagnetic field near the interface which plays the role of a flat resonator. The TS lies in the forbidden frequency gap of the MPC and reveals itself as a very narrow peak of the wave transmission coefficient (transmission zone) through the finite system (the MPC bounded with an NPM).

The development of miniature electronically controlled elements for GHz and THz bands with the use of an MPC is a very actual problem now. In particular, finite volume structures with very narrow frequency transmission zones such as TS are of special interest. The possibility of tuning and changing parameters of the transmission zone is a problem of prime importance.

As for the TS the width and the frequency position of the transmission zone depend both on the properties of the MPC and on external parameters such as characteristics of the NPM and the external applied magnetic field. We can not only control the system by changing the external parameters, but we can also find the NPM characteristics by solving inverse problem, using the experimental frequency TS position.

In this work a system consisting of an MPC and an NPM was studied both experimentally and theoretically in the microwave frequency band. The NPM was prepared as a set of conductive wires (wire medium (WM)). The MPC three-layer elementary cell comprises air, ferrite and quartz layers. A change in the TS spectral position was calculated theoretically for two cases: depending on the thickness of the air layer in the MPC elementary cell and on the value of the NPM. Using the experimental frequency position of the TS we solved the inverse problem and found the effective permittivity of the medium bounding the MPC. Therefore, a method for measuring the value of the medium effective permittivity on the basis of the TS frequency position was proposed.

Note that the three-layer structure of the MPC elementary cell considered here for the first time seems to have considerable potential from the standpoint of further miniaturization of devices involving the MPC. Really, as a rule, the magnetic layers are in this case magnetic thin films on the substrate and for coarse initial tuning of the system the possibility of mechanical rearrangement (displacement)

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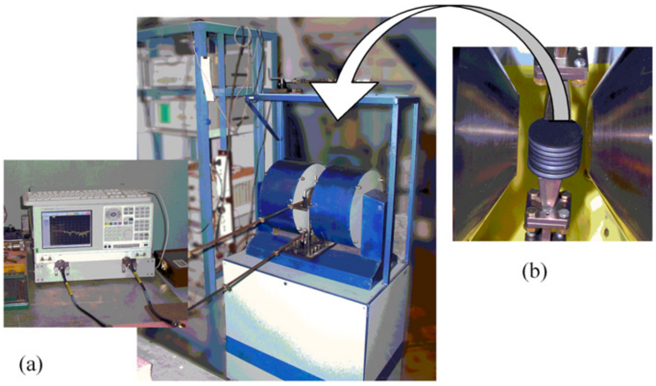

Figure 1. Photo of the test bench (*a*) and a typical MPC-structure between the poles of the magnet (*b*).

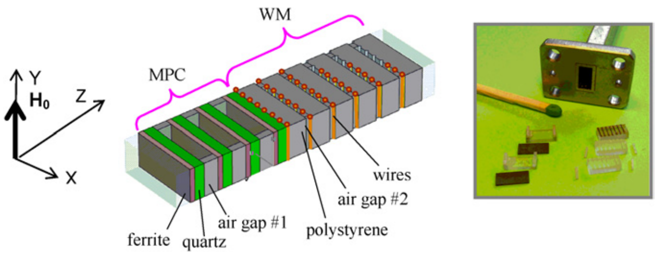

Figure 2. Scheme (*a*) and photo (*b*) of the structure under study.

of such elements should be allowed. This leads necessarily to a three-layer elementary cell. New effects specific for nanostructures and thin magnetic films such as giant magnetoresistance and giant magnetoimpedance can also become operational in such systems as they require far fewer external fields to change constitutive parameters such as permeability and conductivity [5, 6].

2. Specimen and experimental details

The scheme of the experiment presented in figures 1(*a*) and (*b*) is similar to the one described in [7]. A vector network analyzer 'Agilent N5230A' was used as a generator and detector of radiation. The structure under study (figure 2(*a*, *b*)) was connected to the network analyser by a conventional waveguide transmission line.

A static magnetic field H_0 was generated with an electromagnet and changed in the range 0–7000 Oe. Its measurement error did not exceed 1%. The inhomogeneity of the external magnetic field in the volume of magnetic layers did not exceed 1% and did not appreciably influence the observed effects. The studied structure was placed in a gap of the electromagnet, the static magnetic field H_0 being perpendicular both to the wave vector (along the z axis) and the vector of the magnetic wave component (along the x axis). With this field configuration the extraordinary wave propagated in ferrite. The wave transmission coefficient was measured in the frequency band 22–40 GHz.

The specimen under study (figures 2(*a*) and (*b*)) represents a 1D MPC placed into a section of a single-mode waveguide of $7.2 \times 3.4\,\text{mm}^2$. The structure consisting of a sequence (four cells) of three-layer elementary cells is in contact with a WM. The WM is a set of thin wires on the polystyrene substrate, the distance between the wires being much less than the waveguide wave length.

The characteristics of the WM are as follows:

(a) dimensions of the set of copper wires: diameter of the wire is 0.15 mm, the distance between the wires is 1 mm (maximum eight wires in a layer);
(b) polystyrene: thickness $d_{\text{p}} = 2.1$ mm, $\varepsilon' = 4, 9$, $\varepsilon'' = 10^{-3}$;
(c) air gap #2: thickness $d_{\text{a2}} = 0.5$ mm.

Every elementary cell of the MPC consists of three layers: air–ferrite–quartz placed in tandem. The MPC layers parameters are as follows.

(a) Air gap #1: thickness $d_{\text{a1}} = 1.5$ mm.
(b) Quartz layer: thickness $d_{\text{q}} = 1$ mm, $\varepsilon' = 4.5$, $\varepsilon'' = 10^{-4}$;
(c) Ferrite (brand 1SCH4): layer thickness $d_{\text{f}} = 0.5$ mm, $\varepsilon' = 11.1$, $\varepsilon'' = 4 \times 10^{-3}$–$2 \times 10^{-3}$; tensor ferrite permeability $\hat{\mu}$ is defined from the magnetic susceptibility equation for the 22–40 GHz frequency band and has the form (1). Note that the effective permeability for the extraordinary wave is given by expression (2).

3. TS equations

We consider the MPC with a three-layer elementary cell. Two layers (quartz and air) are nonmagnetic materials with constant scalar permittivity and permeability ε_i, μ_i ($i = 1, 2$). As for ferrite (the third layer), which is also isotropic in the absence of the external applied magnetic field, in the magnetic field it becomes magnetized and acquires the magnetization $\vec{M}_0$ ($\vec{M}_0 || \vec{H}_0$).

Now the tensor of ferrite permeability takes the form

$$\hat{\mu} = \begin{pmatrix} \mu & \mathrm{i}\mu_a & 0 \\ -\mathrm{i}\mu_a & \mu & 0 \\ 0 & 0 & \mu_{\|} \end{pmatrix}. \quad (1)$$

Here $\mu_{\|}$ is a tensor component in the $\vec{M}_0$ direction. The ferrite permittivity as before is supposed to be scalar and constant.

As is known [8], the so-called extraordinary wave can propagate in magnetized ferrite in the direction perpendicular to $\vec{M}_0$. The wave electric vector $\vec{E}$ is directed along $\vec{M}_0$. And the plane of the wave vector $\vec{k}$ and the wave magnetic component vector $\vec{H}$ is perpendicular to the magnetization $\vec{M}_0$. The effective permeability of the extraordinary wave equals

$$\mu_{\text{ef}} = \frac{\mu^2 - \mu_{\text{a}}^2}{\mu}. \quad (2)$$

Note that μ and μ_{a} depend on the external magnetic field. For ferrite 1SCH4 the field dependence of μ_{ef} was calculated in [7] correspondingly for the non-saturated ($H_0 < 1000$ Oe) and the saturated cases ($H_0 > 1800$ Oe).

According to the above, if the external magnetic field is perpendicular both to the axis of the crystal (z axis) and to the wave magnetic component vector (see figure 2(a)) the extraordinary wave will propagate along the crystal axis.

To derive the equation for the Bloch wave number we consider the TE wave propagating along the axis of the periodical structure and use the method of the 2×2 transition matrix (see, for example, [9]). Fourier components E_y and H_x in the ith layer ($i = 1, 2, 3$) are related by

$$H_x = \frac{j \cdot c}{\mu_i \omega} \frac{\partial}{\partial z} E_y = -\varsigma_i E_y,$$

$$\text{where} \quad E_y = A_i \mathrm{e}^{j \cdot k_{zi}(z - z_i)} + B_i \mathrm{e}^{-j \cdot k_{zi}(z - z_i)} \tag{3}$$

$$\text{and} \quad \varsigma_i = \frac{k_{zi} c}{\mu_i \omega}, \quad k_{zi} = \sqrt{\varepsilon_i \mu_i \frac{\omega^2}{c^2} - k_x^2}.$$

(Due to uniformity in x the x-dependence is proportional to $\exp(j \cdot k_x x)$ with constant k_x.)

The transition matrix $\hat{M}$ for the elementary cell is the product of the layer transition matrices and for the three-layer cell the corresponding unimodular matrix $\hat{M}$ has the form

$$\hat{M} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}, \tag{4}$$

where the matrix components m_{ik} are

$$\begin{aligned} m_{11} &= \cos\alpha_3 \cos\alpha_2 \cos\alpha_1 - \frac{\varsigma_1}{\varsigma_2} \cos\alpha_3 \sin\alpha_2 \sin\alpha_1 \\ &\quad - \frac{\varsigma_2}{\varsigma_3} \sin\alpha_3 \sin\alpha_2 \cos\alpha_1 - \frac{\varsigma_1}{\varsigma_3} \sin\alpha_3 \cos\alpha_2 \sin\alpha_1, \\ m_{12} &= -\frac{\mathrm{i}}{\varsigma_1} \cos\alpha_3 \cos\alpha_2 \sin\alpha_1 - \frac{\mathrm{i}}{\varsigma_2} \cos\alpha_3 \sin\alpha_2 \cos\alpha_1 \\ &\quad + \frac{\mathrm{i}\varsigma_2}{\varsigma_1 \varsigma_3} \sin\alpha_3 \sin\alpha_2 \sin\alpha_1 - \frac{\mathrm{i}}{\varsigma_2} \sin\alpha_3 \cos\alpha_2 \cos\alpha_1, \\ m_{21} &= -\mathrm{i}\varsigma_3 \sin\alpha_3 \cos\alpha_2 \cos\alpha_1 + \frac{\mathrm{i}\varsigma_3 \varsigma_1}{\varsigma_2} \sin\alpha_3 \sin\alpha_2 \sin\alpha_1 \\ &\quad - \mathrm{i}\varsigma_2 \cos\alpha_3 \sin\alpha_2 \cos\alpha_1 - \mathrm{i}\varsigma_1 \cos\alpha_3 \cos\alpha_2 \sin\alpha_1, \\ m_{22} &= -\frac{\varsigma_3}{\varsigma_1} \sin\alpha_3 \cos\alpha_2 \sin\alpha_1 - \frac{\varsigma_3}{\varsigma_2} \sin\alpha_3 \sin\alpha_2 \cos\alpha_1 \\ &\quad - \frac{\varsigma_2}{\varsigma_1} \cos\alpha_3 \sin\alpha_2 \sin\alpha_1 + \cos\alpha_3 \cos\alpha_2 \cos\alpha_1, \end{aligned} \tag{5}$$

where $\alpha_i = k_{zi} d_i$, here index i ($i = 1, 2, 3$) is the number of the layer .

Floquet's theorem accounts for the periodicity of the MPC:

$$\hat{M} \begin{pmatrix} E_y(0) \\ H_x(0) \end{pmatrix} = \mathrm{e}^{\mathrm{i}k_{\mathrm{B}} d} \begin{pmatrix} E_y(0) \\ H_x(0) \end{pmatrix}, \tag{6}$$

where k_{B} is the Bloch wave number and $d = d_1 + d_2 + d_3$ is the period of the MPC. Using equation (2) and relations (5) we obtain the dispersion equation for the Bloch wave number in the form

$$\begin{aligned} \cos k_{\mathrm{B}} d &= \cos\alpha_3 \cos\alpha_2 \cos\alpha_1 \\ &\quad - \frac{1}{2}\left(\frac{\varsigma_1}{\varsigma_2} + \frac{\varsigma_2}{\varsigma_1}\right) \cos\alpha_3 \sin\alpha_2 \sin\alpha_1 \\ &\quad - \frac{1}{2}\left(\frac{\varsigma_1}{\varsigma_3} + \frac{\varsigma_3}{\varsigma_1}\right) \sin\alpha_3 \cos\alpha_2 \sin\alpha_1 \\ &\quad - \frac{1}{2}\left(\frac{\varsigma_2}{\varsigma_3} + \frac{\varsigma_3}{\varsigma_2}\right) \sin\alpha_3 \sin\alpha_2 \cos\alpha_1. \end{aligned} \tag{7}$$

The dispersion equation for the TS follows from the equality of the MPC

$$\left(Z = -\frac{H_x(0)}{E_y(0)}\right)$$

and the bounding medium $\left(-\sqrt{\frac{\varepsilon}{\mu}}\right)$ impedances [3], namely,

$$\sqrt{\frac{\varepsilon}{\mu}} = \left.\frac{m_{21} - \varsigma_2(m_{11} - \mathrm{e}^{j \cdot k_{\mathrm{B}} d})}{m_{22} - \mathrm{e}^{j \cdot k_{\mathrm{B}} d} - \varsigma_2 m_{12}}\right|_{k_x = 0}, \tag{8}$$

where components m_{ik} are given with expressions (5). The essential feature of the TS lies in the fact that the component k_x is zero. This implies that the TS is not connected with the energy transfer along the interface (MPC–WM). So we should put $k_x = 0$ in expressions (5) for m_{ik} which enter equation (8) in the form

$$\alpha_i|_{k_x=0} = \frac{\omega}{c} d_i \sqrt{\varepsilon_i \mu_i}, \qquad \varsigma_i|_{k_x=0} = \frac{c}{\omega}\sqrt{\frac{\varepsilon_i}{\mu_i}}. \tag{9}$$

Equation (8) determines the TS frequency, which depends both on the internal MPC parameters (thicknesses of layers, permittivities and permeabilities of layers) and the external parameters such as the applied magnetic field, permittivity and permeability of the WM. Therefore, if the TS frequency is given, we can determine some of these parameters, in particular, the permittivity of a finite WM layer. This characteristic can be important if we consider a photonic crystal which includes layers with negative permittivity.

The TS has a very important special feature. As was shown in [3], a sharp narrow peak appears in the transmission spectrum as a result of resonant tunnelling of the electromagnetic wave through the TS. This peak is located in the band gap of the MPC at the TS frequency (see figure 4).

In this work we obtained numerical solutions of equation (8), which determined the TS position at various values of parameters. The curve shown in figure 3 describes the TS position depending on the change in the air MPC layer thickness.

The dependence in figure 3 was calculated at a constant WM permittivity equal to $\varepsilon_{\mathrm{WM}} = -2, 77$.

We see that the Tamm peak frequency position is sensitive to the change in the air gap, so this quantity can be a good parameter to control and change the position of the sharp peak in the transmission spectrum. The solid curve in figure 5 shows the dependence of the Tamm peak frequency position on the value of the WM permittivity when all the MPC parameters are fixed.

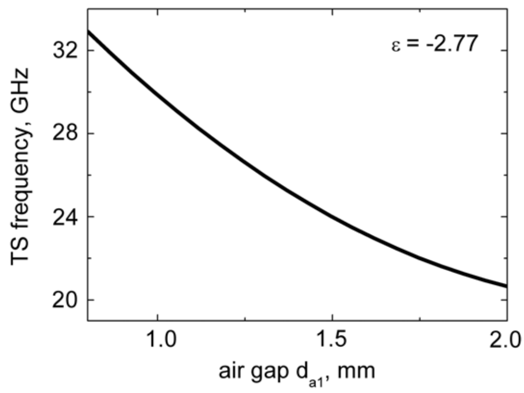

Figure 3. Dependence of the Tamm peak position on the value of the air layer thickness d_{a1}.

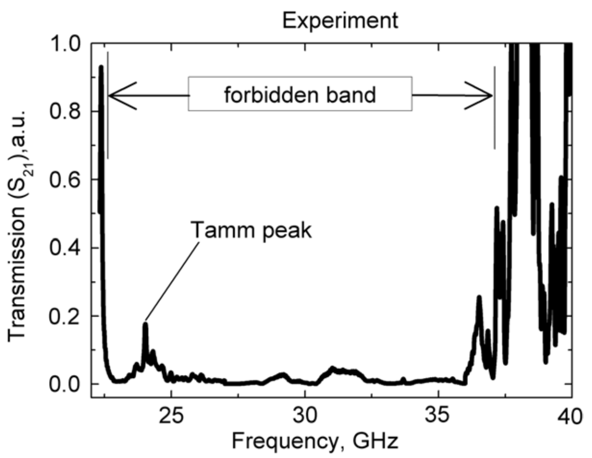

Figure 4. Experimental spectrum demonstrating the formation of the Tamm peak in the forbidden zone of the MPC for $H_0 = 1880$ Oe.

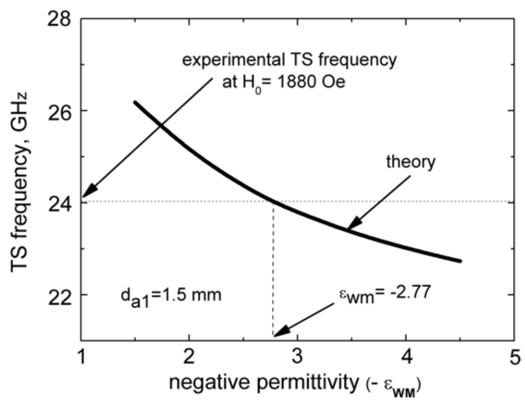

Figure 5. Variation of the TS frequency with the change in the WM permittivity $\varepsilon_{\mathrm{WM}}$ for the magnetic field $H_0 = 1880$ Oe at $d_{a1} = 1.5$ mm.

4. Experimental results and discussions

We have experimentally measured the transmission coefficient S_{21} through the system (MPC + WM). According to figure 4 the narrow peak, which corresponds to the TS, exists in the forbidden band of the MPC. Its frequency is $f_{\mathrm{TS}} = 24.030$ GHz. The peak's Q factor is about 10^3, which is a rather high magnitude for oscillations excited in the waveguide.

With the help of the measured experimental TS frequency and figure 5 we can solve the inverse problem and find the value of WM permittivity. As can be seen from figure 5, this value equals in this case $\varepsilon_{\mathrm{WM}} = -2,77$. Using this value of WM permittivity we can solve equation (8) and find the change in the TS frequency position depending on the change, for example, in the air layer thickness (figure 3).

Note here that now the problem of WM permittivity is actually in particular for the theory of the so-called left handed media and is the subject of considerable literature [10–12]. There are a number of formulae for permittivity based on plasma theory where the mass of carriers is connected with the parameters of the WM. One such well-known formula has the form [10]

$$\varepsilon_{\mathrm{WM}}(\omega) = \varepsilon_{\mathrm{h}} - \frac{2\pi c^2}{\omega^2 a^2 \ln(a/r)}, \tag{10}$$

where a is the distance between the wires and r is the wire radius. ε_h is the permittivity of the surrounding (host) medium. This formula describes the permittivity of an infinite periodical WM.

The experimental TS frequency position gives the answer to the question about the magnitude of permittivity $\varepsilon_{\mathrm{WM}}$ for a finite WM layer. According to our findings (figure 5), the WM permittivity equals $\varepsilon_{\mathrm{WM}} = -2.77$. Therefore, formula (10) overestimates the permittivity of the WM for the given frequency of the TS ($\varepsilon_{\mathrm{WM}} = -4.68$) for the host medium—polystyrene.

5. Conclusions

1. The theoretical model of a 1D MPC was developed for the case of a three-layer elementary cell. The dispersion equation for the TS was obtained and analysed numerically.
2. The experimental study of the MPC adjoining the WM was provided. The TP position in the forbidden zone of the MPC corresponds to the frequency $f = 24.03$ GHz at the external magnetic field of $H = 1880$ Oe.
3. The change in the TS frequency position depending both on the change in the MPC structural parameter (d_{a1}) and on WM permittivity was studied theoretically.
4. The comparison of the experimental TS frequency with the results of calculations made it possible to determine the effective permittivity of the finite WM layer and to calculate the dependence of the TP frequency position on the thickness of the MPC air layer. Consequently, the experimental determination of the TS frequency position may serve as a method for the measurement of the WM effective permittivity.

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