Pulsed Radiation From a Line Electric Current Near a Planar Interface: A Novel Technique

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***Abstract*—A novel technique has been suggested for the analysis of a transient electromagnetic field generated by a pulsed line current that is located near a planar interface between two dielectric nonabsorbing and nondispersive media. As distinct from the Cagniard-de Hoop method, which is widely used for the study of transient fields both in electrodynamics and in the theory of acoustic and seismic waves, our approach is based on the transformation of the domain of integration in the integral expression for the field in the space of two complex variables. As a result, it will suffice to use the standard procedure of finding of roots of the algebraic equation rather than construct auxiliary Carniard's contours. A fresh type of the representation for the field has been derived in the form of an integral along a finite contour. The algorithm based on the representation of this kind may work as the most efficient tool for calculating fields in multilayered media. The method suggested allows extension to the case of arbitrary dipole sources.**

***Index Terms*—Electromagnetic radiation, modified Cagniard technique, planar interface, pulsed line source.**

I. Introduction

TRANSIENT electromagnetic fields generated by pulsed currents located near a planar boundary between layered media are the subject of constant theoretical research, as from the B. van der Pol paper [1]. The approach based on the classical Cagniard method [2], [3] is the most efficient tool in this study. De Hoop [4] has suggested a modification of Cagniard's method with the help of which exact solutions have been obtained for a number of problems about a dipole or a line source near an interface [5]–[9].

Various modifications of Cagniard's technique have found wide application in the study of nonstationary acoustic and seismic wave propagation. Following paper [4], modifications of de Hoop's technique [10], [11] as well as the alternative approaches free from some drawbacks to this method [12], [13] have been suggested.

In the present paper, the approach alternative to Cagniard's technique is used to study the nonstationary field generated by line sources located in flat-layered media. The approach suggested is applied to the already solved problem, namely, determination of the electromagnetic field generated by a pulsed line source located near a planar interface between two nonabsorbing and nondispersive media. The corresponding results have been discussed in considerable detail in [9]. In this paper, a one-sided Laplace transform with respect to time and two-sided Laplace transform with respect to a horizontal spatial variable have been applied and, as a consequence, the electromagnetic field has been represented in the form of some double integral. This integral can be calculated efficiently by the Cagniard-de Hoop method (CHM). The essence of the method is as follows. The original path of integration for one of two integrals forming the double integral is deformed into a so-called modified Cagniard contour. It is chosen such that upon the corresponding change of the integration variable in the integral along the modified contour, the original double integral turns into a composition of the direct and inverse Laplace transform for the known function. The central problem with this method is finding, generally speaking, numerically, the modified Cagniard contour whose shape changes as the observation point changes.

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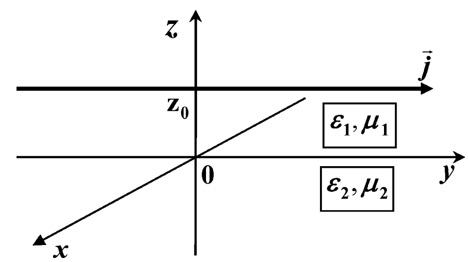

Fig. 1. Pulsed line source near the interface between two semi-infinite media.

The key point of the approach proposed in the present paper includes the following. To calculate the double integral efficiently, we suggest deforming its domain of integration (the real plane) in the $\mathbf{C}^2$-space of two complex variables rather than to deform one contour in the complex $\mathbf{C}^1$-plane, as has been done in CHM. It is shown that in this case the integral reduces to a sum of residues. The use of powerful apparatus of the residue theory instead of somewhat artificial way used in CHM is reason to hope that our approach can be efficient in the situations where the CHM fails, such as for anisotropic media. The method presented in the paper can be extended to multilayered media and arbitrary dipole sources.

II. Problem Formulation

The field generated by the pulsed line electric current

$$j_y = I_0\delta(x)\delta(z-z_0)\delta(t) \quad (z_0>0) \tag{1}$$

located near a planar interface (Fig. 1) is to be found. The source of this kind excites the E-polarized field

$$\begin{aligned} &E_y \neq 0, \;\; E_x = E_z = 0 \\ &\frac{\partial H_x}{\partial t} = \mu^{-1}\frac{\partial E_y}{\partial z}, \;\; \frac{\partial H_z}{\partial t} = -\mu^{-1}\frac{\partial E_y}{\partial x} \end{aligned} \tag{2}$$

where $\varepsilon=\varepsilon_1, \mu=\mu_1$ for $z>0$ and $\varepsilon=\varepsilon_2, \mu=\mu_2$ for $z<0$.

The function E_y is the solution to the wave equation

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}-\varepsilon\mu\frac{\partial^2}{\partial t^2}\right)E_y=\mu_1\frac{\partial j_y}{\partial t} \tag{3}$$

that satisfies the conditions of continuity of E_y- and H_x-components on the interface and the causality principle.

The Fourier transform in time

$$F(x,z;\omega)=\frac{1}{2\pi}\int\limits_{-q\infty}^{\infty}E_y(x,z;t)e^{i\omega t}dt$$
$$E_y(x,z;t)=\int\limits_{-\infty}^{\infty}F(x,z;\omega)e^{-i\omega t}d\omega \tag{4}$$

applied to the boundary-value problem in (3), results in the following problem

$$\begin{aligned}&\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}+\omega^2\varepsilon_1\mu_1\right)F^1=-g_0\delta(x)\delta(z-z_0)\quad(z>0)\\&\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}+\omega^2\varepsilon_2\mu_2\right)F^2=0\qquad(z<0)\end{aligned} \tag{5}$$

with the boundary conditions on $z=0$

$$1)\ F^1=F^2,\quad 2)\ \mu_2\frac{\partial F^1}{\partial z}=\mu_1\frac{\partial F^2}{\partial z} \tag{6}$$

where $g_0=i\omega\mu_1 I_0/2\pi$. The solution of the equations in (5) is conveniently represented in the form [7]

$$\begin{aligned}F^1&=g_0\left(F^0+F_s^1\right)\qquad(z>0)\\F^2&=g_0F_s^2\qquad(z<0)\end{aligned} \tag{7}$$

where

$$F^0=\frac{i}{4\pi}\int\limits_{-\infty}^{\infty}\frac{\exp\left\{i\xi x+i\sqrt{k_1^2-\xi^2}|z-z_0|\right\}}{\sqrt{k_1^2-\xi^2}}d\xi=\frac{1}{4}H_0^{(1)}(k_1R_-) \tag{8}$$

$$F_s^1=\frac{i}{4\pi}\int\limits_{-\infty}^{\infty}\frac{\exp\left\{i\xi x+i\sqrt{k_1^2-\xi^2}(z+z_0)\right\}}{\sqrt{k_1^2-\xi^2}}\times\Gamma_1(\xi,\omega)d\xi \tag{9}$$

$$F_s^2=\frac{i}{4\pi}\int\limits_{-\infty}^{\infty}\frac{\exp\left\{i\xi x-i\sqrt{k_2^2-\xi^2}z+i\sqrt{k_1^2-\xi^2}z_0\right\}}{\sqrt{k_1^2-\xi^2}}\times\Gamma_2(\xi,\omega)d\xi \tag{10}$$

$\Gamma_j(\xi,\omega)$ are the unknown functions, $\mathrm{Im}\sqrt{k_j^2-\xi^2}\geq 0, k_j^2=\omega^2n_j^2, n_j^2=\varepsilon_j\mu_j,(j=1,2), R_-^2=x^2+(z-z_0)^2$. From the boundary conditions in (6), we have:

1) $1+\Gamma_1=\Gamma_2$.
2) $-\mu_2\sqrt{k_1^2-\xi^2}+\mu_1\sqrt{k_1^2-\xi^2}\Gamma_1=-\mu_1\sqrt{k_2^2-\xi^2}\Gamma_2$

or

$$\Gamma_1(\xi,\omega)=\frac{\mu_2\sqrt{k_1^2-\xi^2}-\mu_1\sqrt{k_2^2-\xi^2}}{\mu_2\sqrt{k_1^2-\xi^2}+\mu_1\sqrt{k_2^2-\xi^2}} \tag{11}$$

$$\Gamma_2(\xi,\omega)=\frac{2\mu_2\sqrt{k_1^2-\xi^2}}{\mu_2\sqrt{k_1^2-\xi^2}+\mu_1\sqrt{k_2^2-\xi^2}}. \tag{12}$$

Thus, we have the required field in the form of the following double integrals over the plane P of real variables ω and ξ

$$E_y^j(x,z;t)=E_0\frac{\partial}{\partial t}G^j(x,z;t),\quad(j=0,1,2) \tag{13}$$

$$G^0(x,z;t)=\frac{1}{4\pi i}\iint\limits_P\exp\left\{i\xi x+i\sqrt{\omega^2n_1^2-\xi^2}\times|z-z_0|-i\omega t\right\}\frac{d\omega d\xi}{\sqrt{\omega^2n_1^2-\xi^2}}\quad(z>0), \tag{14}$$

$$G^1(x,z;t)=\frac{1}{4\pi i}\iint\limits_P\exp\left\{i\xi x+i\sqrt{\omega^2n_1^2-\xi^2}\times(z+z_0)-i\omega t\right\}\frac{\Gamma_1(\xi,\omega)d\omega d\xi}{\sqrt{\omega^2n_1^2-\xi^2}}\quad(z>0), \tag{15}$$

$$G^2(x,z;t)=\frac{1}{4\pi i}\iint\limits_P\exp\left\{i\xi x-i\sqrt{\omega^2n_2^2-\xi^2}z+i\sqrt{\omega^2n_1^2-\xi^2}z_0-i\omega t\right\}\cdot\frac{\Gamma_2(\xi,\omega)d\omega d\xi}{\sqrt{\omega^2n_1^2-\xi^2}}\quad(z<0) \tag{16}$$

where $E_0=I_0\mu_1/2\pi$, $\mathrm{Im}\sqrt{\omega^2n_j^2-\xi^2\geq 0}$ $(j=1,2)$.

III. Transformation to Single Integrals

In formulas (14)–(16), the integrands allow analytic continuation from the real plane $P=\{\omega,\xi;\omega''=\xi''=0\}$ into the $\mathbf{C^2}$-space of two complex variables $\omega=\omega'+i\omega''$ and $\xi=\xi'+i\xi''$. As previous analysis has shown, there is no need to operate with the whole of real 4-D space $\mathbf{C^2}$. To calculate efficiently the integrals in (14)–(16), it is sufficient to consider a 3-D space $\mathbf{R^3}=\{\omega,\xi;,\xi''=0\}\subset\mathbf{C^2}$ containing P. In $\mathbf{R^3}$, the single-valued branches of two square roots in the integrands should be chosen. In Appendix, it is shown that for a loss-free media the cut surface ensuring a choice of the branch for which we have $\mathrm{Im}\sqrt{\omega^2n-j^2-\xi^2\geq 0}$ in $\mathbf{R^3}$ is (Fig. 4) a double sector S_j that lies in the plane $\omega''=0$, contains the ω'-axis, and is bounded by the branch lines $n_j\omega'\pm\xi'=0$ $(j=1,2)$. The root is positive on the upper side of the right-hand sector $(\omega'>0,\omega''=0+0)$ and on the bottom side of the left-hand sector $(\omega'<0,\omega''=0-0)$; while it is negative on the other sides. Since the integrands in (14)–(16) are uniquely defined in the $\mathbf{R^3}$-space with the specified cuts, the Cauchy-Poincare theorem [14] can be used to deform the surface of integration P in $\mathbf{R^3}\setminus(S_1\cup S_2)$.

In accordance with the causality principle, the cut surfaces S_1 and S_2 have to adjoin the real plane P from the bottom ($\omega''=$

$0-0$). Then, the integrands have no singularities in the half-space $\omega'' > 0$, and we have $E_y(x,z;t) \equiv 0$ for all $t < 0$, according the mentioned theorem.

For positive values of the time variable t, the P-plane can be deformed to a half-space $\omega'' < 0$. Then we have for E_y^0 an integral over the surface P_{c1}, while for E_y^1, E_y^2 we have integrals over the surface $P_c = P_{c1} \cup P_{c2}$. Here P_{cj} stands for the closed surface enveloping the cut S_j.

Using the function $G^1(x,z;t)$ as an example, let us demonstrate how the integrals describing the secondary field in (15), (16) can be simplified. Denoting the integrand in (15) by $f(\omega,\xi')$, consider the following integral over the surface P_c

$$\iint\limits_{P_c} f(\omega,\xi')ds = I_1 + I_2 \tag{17}$$

where $I_j = \int\int_{P_{cj}} f(\omega,\xi')ds$. Let P_{cj}^+ and P_{cj}^- be the right-hand ($\omega' > 0$) and the left-hand ($\omega' < 0$) cavities of the surface P_{cj}; $L_{\omega' j}$ is the closed contour generated by the intersection of the surface P_{cj} with the coordinate plane $\omega' = const (j = 1, 2)$. Then we have

$$\begin{aligned} I_1 &= \sum_{\pm} \iint\limits_{P_{c1}^{\pm}} f(\omega,\xi')ds \\ &= \int_0^\infty d\omega' \int\limits_{L_{\omega'1}} dl f(\omega,\xi') + \int_{-\infty}^0 d\omega' \int\limits_{L_{\omega'1}} dl f(\omega,\xi') \\ &= \int_0^\infty d\omega' \left[\int\limits_{L_{\omega'1}} dl f(\omega,\xi') - \int\limits_{L_{\omega'1}} dl f(-\omega,-\xi') \right]. \end{aligned} \tag{18}$$

In the second integral here, the change of variables $\omega \to -\omega, \xi' \to -\xi'$ has been carried out. By taking into account the evenness of the chosen branches of square roots entering the function (ω,ξ') with respect to this change of variables and by performing another change of variables $\xi' = \omega\eta$, we arrive at the following expression for the integral in (15)

$$\begin{aligned} I_1 = \int_0^\infty d\omega \int\limits_{L_1} d\eta \Bigg(&\exp\left\{ i\omega \left[\eta x + \sqrt{n_1^2-\eta^2}(z+z_0) - t \right] \right\} \\ &- \exp\Bigg\{ i\omega \Big[-\eta x + \sqrt{n_1^2-\eta^2} \\ &\times (z+z_0) + t \Big] \Bigg\} \Bigg) \Gamma_1'(\eta) / \sqrt{n_1^2-\eta^2} \end{aligned} \tag{19}$$

where

$$\Gamma_1'(\eta) = \frac{\mu_2\sqrt{n_1^2-\eta^2} - \mu_1\sqrt{n_2^2-\eta^2}}{\mu_2\sqrt{n_1^2-\eta^2} + \mu_1\sqrt{n_2^2-\eta^2}} \tag{20}$$

and the contour L_1 envelopes the segment $(-n_1, n_1)$ in the plane of the complex variable η. Let us introduce an accessory parameter $\delta > 0$ for the sake of convergence acceleration; then (19) can be rewritten in the form of (21), shown at the bottom of the page.

For the second integral I_2 in (17), we obtain a representation similar to (21) with L_1 replaced by L_2, where L_2 is the contour enveloping the segment $(-n_2, n_2)$. Thus, for the function given by (15), which determines the secondary field in the first medium (see (13)), we arrive at the following expression:

$$\begin{aligned} G^1(x,z;t) = \frac{1}{4\pi} \lim_{\delta\to 0} \int\limits_L \Bigg[&\frac{1}{\eta x + \sqrt{n_1^2-\eta^2}(z+z_0) - t_-} \\ &- \frac{1}{-\eta x + \sqrt{n_1^2-\eta^2}(z+z_0) + t_+} \Bigg] \frac{\Gamma_1'(\eta)d\eta}{\sqrt{n_1^2-\eta^2}} \end{aligned} \tag{22}$$

where $t_\pm = t \pm i\delta$, L is the contour enveloping the segment $(-n_{\max}, n_{\max})$, $n_{\max} = \max(n_1, n_2)$. The root branches are determined by the inequalities $-\pi < \arg\sqrt{n_j^2-\eta^2} < \pi$ with zero value of the argument on the bottom side of the cut along the segment $(-n_j, n_j)$.

Similarly, for the function G^2 describing the field in the second medium, we obtain from (16)

$$\begin{aligned} G^2&(x,z;t) \\ &= \frac{1}{4\pi} \lim_{\delta\to 0} \int\limits_L \Bigg[\frac{1}{\eta x + \sqrt{n_1^2-\eta^2}z_0 - \sqrt{n_2^2-\eta^2}z - t_-} \\ &\qquad - \frac{1}{-\eta x + \sqrt{n_1^2-\eta^2}z_0 - \sqrt{n_2^2-\eta^2}z + t_+} \Bigg] \\ &\qquad \times \frac{\Gamma_2'(\eta)d\eta}{\sqrt{n_1^2-\eta^2}} \end{aligned} \tag{23}$$

where $\Gamma_2'(\eta) = 1 + \Gamma_1'(\eta)$. The integrands in (22) and (23) are analytical in the plane of complex variable η with the specified cut and decrease at infinity as η^{-2}. Therefore, these integrals

$$\begin{aligned} I_1 = \lim_{\delta\to 0} \int_0^\infty d\omega \int\limits_{L_1} d\eta \Bigg(&\exp\left\{ i\omega \left[i\delta + \eta x + \sqrt{n_1^2-\eta^2}(z+z_0) - t \right] \right\} \\ &- \exp\left\{ i\omega \left[i\delta - \eta x + \sqrt{n_1^2-\eta^2}(z+z_0) + t \right] \right\} \Bigg) \Gamma_1'(\eta) / \sqrt{n_1^2-\eta^2} \\ = i \lim_{\delta\to 0} \int\limits_{L_1} \Bigg[&\frac{1}{\eta x + \sqrt{n_1^2-\eta^2}(z+z_0) - t + i\delta} - \frac{1}{-\eta x + \sqrt{n_1^2-\eta^2}(z+z_0) + t + i\delta} \Bigg] \frac{\Gamma_1'(\eta)d\eta}{\sqrt{n_1^2-\eta^2}} \end{aligned} \tag{21}$$

can be reduced to the residues determined by zero values of the denominators in the square brackets

$$\begin{aligned} a)&\quad \eta x+\sqrt{n_1^2-\eta^2}(z+z_0)-t_-=0\\ b)&\quad -\eta x+\sqrt{n_1^2-\eta^2}(z+z_0)+t_+=0 \end{aligned}\tag{24}$$

$$\begin{aligned} a)&\quad \eta x+\sqrt{n_1^2-\eta^2}z_0-\sqrt{n_2^2-\eta^2}z-t_-=0\\ b)&\quad -\eta x+\sqrt{n_1^2-\eta^2}z_0-\sqrt{n_2^2-\eta^2}z+t_+=0. \end{aligned}\tag{25}$$

IV. Field in the First Medium

The roots of the (24) are readily determined and can be written as

$$\eta_1^-=\left(xt_- -(z+z_0)\sqrt{n_1^2R_+^2-t_-^2}\right)R_+^{-2}\tag{26}$$

and

$$\eta_1^+=\left(xt_+ +(z+z_0)\sqrt{n_1^2R_+^2-t_+^2}\right)R_+^{-2}\tag{27}$$

for (24a) and (24b) respectively, where $R_+^2=x^2+(z+z_0)^2$. For the square root $\sqrt{n_1^2R_+^2-t^2}$, the same branch in the complex plane of variable t has been determined as for $\sqrt{n_j^2-\eta^2}$ in the η-plane. By calculating the corresponding residues, we have from (22)

$$\begin{aligned} G^1(x,z;t)=&\frac{i}{2}\lim_{\delta\to 0}\left\{\left.\frac{\Gamma_1'(\eta)}{x\sqrt{n_1^2-\eta^2}-(z+z_0)\eta}\right|_{\eta=\eta_1^-}\right.\\ &\left.+\left.\frac{\Gamma_1'(\eta)}{x\sqrt{n_1^2-\eta^2}+(z+z_0)\eta}\right|_{\eta=\eta_1^+}\right\}\\ =&\frac{i}{2}\lim_{\delta\to 0}\left\{\frac{\Gamma_1'\left(\eta_1^-\right)}{\sqrt{n_1^2R_+^2-t_-^2}}+\frac{\Gamma_1'\left(\eta_1^+\right)}{\sqrt{n_1^2R_+^2-t_+^2}}\right\}. \end{aligned}\tag{28}$$

Here we have used the equality

$$\sqrt{n_1^2-\left(\eta_1^{\mp}\right)^2}=\left(x\sqrt{n_1^2R_+^2-(t_{\mp})^2}\pm(z+z_0)t_{\mp}\right)R_+^{-2}.\tag{29}$$

It is easy to verify that the following relationships hold for the chosen branches of the square roots:

$$\begin{aligned} \sqrt{n_1^2R_+^2-(t^*)^2}&=e^{i\pi}\left(\sqrt{n_1^2R_+^2-t^2}\right)^*\\ \sqrt{n_j^2-(\eta^*)^2}&=e^{i\pi}\left(\sqrt{n_j^2-\eta^2}\right)^*. \end{aligned}\tag{30}$$

Therefore

$$\begin{aligned} \eta_1^-&=\left[xt_+^*-(z+z_0)e^{i\pi}\left(\sqrt{n_1^2R_+^2-t_+^2}\right)^*\right]R_+^{-2}\\ &=\left[xt_+ +(z+z_0)\sqrt{n_1^2R_+^2-t_+^2}\right]^*R_+^{-2}\\ &=\left(\eta_1^+\right)^* \end{aligned}$$

$$\begin{aligned} \sqrt{n_j^2-\left(\eta_1^-\right)^2}&=e^{i\pi}\left(\sqrt{n_j^2-\left(\left(\eta_1^-\right)^*\right)^2}\right)^*\\ &=-\left(\sqrt{n_j^2-\left(\eta_1^+\right)^2}\right)^*\\ \Gamma_1'\left(\eta_1^-\right)&=\left(\Gamma_1'\left(\eta_1^+\right)\right)^*. \end{aligned}\tag{31}$$

The wave reflected from the interface comes at the given point in the first medium at time $t_{ref}=n_1R_+$. For the time interval $0<t<t_{ref}$, in view of (31), we obtain

$$\begin{aligned} G^1(x,z;t)&=\frac{i}{2}\lim_{\delta\to 0}\left\{\frac{\Gamma_1'^*\left(\eta_1^+\right)}{\sqrt{n_1^2R_+^2-t_-^2}}+\frac{\Gamma_1'\left(\eta_1^+\right)}{\sqrt{n_1^2R_+^2-t_+^2}}\right\}\\ &=\frac{i}{2}\lim_{\delta\to 0}\left\{\Gamma_1'^*\left(\eta_1^+\right)-\Gamma_1'\left(\eta_1^+\right)\right\}\Big/\sqrt{n_1^2R_+^2-t^2}\\ &=\mathrm{Im}\Gamma_1'\left(\eta_1^<\right)\Big/\sqrt{n_1^2R_+^2-t^2} \end{aligned}\tag{32}$$

where $\eta_1^< = \eta_1^+|_{\delta=0}=(xt-(z+z_0)\sqrt{n_1^2R_+^2-t^2})R_+^{-2}$.

For the time interval $t_{ref}<t$, we have

$$\begin{aligned} G^1(x,z;t)&=\frac{i}{2}\lim_{\delta\to 0}\left\{\frac{\Gamma_1'^*\left(\eta_1^+\right)}{i\sqrt{t^2-n_1^2R_+^2}}+\frac{\Gamma_1'\left(\eta_1^+\right)}{i\sqrt{t^2-n_1^2R_+^2}}\right\}\\ &=\mathrm{Re}\Gamma_1'\left(\eta_1^>\right)\Big/\sqrt{t^2-n_1^2R_+^2} \end{aligned}\tag{33}$$

where $\eta_1^> = \eta_1^+|_{\delta=0}=(xt+i(z+z_0)\sqrt{t^2-n_1^2R_+^2})R_+^{-2}$.

The behavior of the secondary field in the first medium for the times $0<t<t_{ref}$ essentially depends on the relation between the refractive indices for the first (n_1) and the second (n_2) media.

For an arbitrary point in the first medium, both of the roots entering in $\Gamma_1'(\eta_1^<)$ are real (see (29)) if $n_1<n_2$. Consequently, we have $\mathrm{Im}\Gamma_1'(\eta_1^<)=0$, and the secondary field given by (32) is zero $(G^1(x,z;t)\equiv 0)$ up to the moment of arrival of the reflected wave.

In the case that $n_1>n_2$, a more detailed analysis of the function $n_2^2-(\eta_1^<)^2$ is required. Let us use the following notations: $x/R_+=\sin\theta$, $(z+z_0)/R_+=\cos\theta$, $n_2/n_1=\sin\theta_{tot}$, where θ_{tot} stands for the angle of total internal reflection [7], [15]. Let also introduce a parameter $\tau=\arccos(t/t_{ref})$, where the principal branch $0<\tau<\pi$ of this function has been chosen, through the formula $\cos\tau=t/t_{ref}$. Then we arrive at

$$\begin{aligned} n_2^2-\left(\eta_1^<\right)^2&=n_1^2\left\{\left[\frac{x}{R_+}\sqrt{1-\left(\frac{t}{n_1R_+}\right)^2}\right.\right.\\ &\qquad\left.\left.+\frac{(z+z_0)}{R_+}\frac{t}{n_1R_+}\right]^2-\left(1-\frac{n_2^2}{n_1^2}\right)\right\}\\ &=n_1^2\left\{\cos^2(\tau-\theta)-\cos^2\theta_{tot}\right\}\\ &=n_1^2\sin(\theta_{tot}-\theta+\tau)\sin(\theta_{tot}+\theta-\tau). \end{aligned}\tag{34}$$

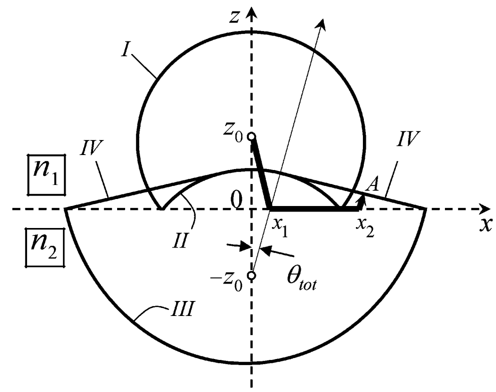

Fig. 2. Wave fronts of the field generated by the line pulsed current located near a planar interface for $n_1 > n_2$: primary (I), reflected (II), transmitted (III), and "side" (IV) waves; $z_0 x_1 x_2 A$ is the trajectory determining the time of arrival of the "side" wave at the point A, θ_{tot} is the angle of total internal reflection.

Since for the space-time domain considered we have $0 < \theta, \theta_{tot}, \tau < \pi/2$, then the arguments of the sine functions in (34) find themselves within the interval $(-\pi/2, \pi)$. Therefore, the function given by (34) has two roots $\tau_1 = \theta - \theta_{tot}$ and $\tau_2 = \theta + \theta_{tot}$ corresponding to the points of time $t_1 = t_{ref}\cos(\theta - \theta_{tot})$ and $t_2 = t_{ref}\cos(\theta + \theta_{tot})$. There is no difficulty to show (the trajectory $z_0 x_1 x_2 A$ in Fig. 2) that $t_1 = n_1 z_0/\cos\theta_{tot} + n_2(x - (z + z_0)tg\theta_{tot}) + n_1 z/\cos\theta_{tot} = t_{dif}$, where t_{dif} is the time of arrival of the so-called 'side' [7] (or 'diffraction' [15]) wave at the observation point located in the first medium in the region $\theta > \theta_{tot}$. For $\theta < \theta_{tot}$, the variable τ_1 goes to the unphysical sheet of the function $\arccos(t/t_{ref})$, and the 'side' wave does not occur in this region. By virtue of the causality principle, for the times $t < t_{dif}$, there is no secondary field and so the other zero (τ_2) is of no importance $(t_2 < t_1)$.

Let us find the value of $sign\{n_2^2 - (\eta_1^<)^2\}$ for $t_{dif} < t < t_{ref}$ in the region $\theta > \theta_{tot}$. Here, the following relationships for the arguments of the sine functions in (32) are valid:

$$\begin{aligned} -\pi/2 < \theta_{tot} - \theta < \theta_{tot} - \theta + \tau < \theta_{tot} - \theta + \tau_1 = 0 \\ 0 < 2\theta_{tot} = \theta_{tot} + \theta - \tau_1 < \theta_{tot} + \theta - \tau < \theta_{tot} + \theta < \pi. \end{aligned}$$

which means that $n_2^2 - (\eta_1^<)^2 < 0$. Considering that $\mathrm{Im}\sqrt{n_1^2 - (\eta_1^<)^2} = 0$, we have $\mathrm{Im}\Gamma_1'(\eta_1^<) \neq 0$.

Thus, for $n_1 > n_2$ and $t_{dif} < t < t_{ref}$, the "side" wave given by the function (32) is generated in the region $\theta > \theta_{tot}$.

From (32), (33), through the use of the substitutions $\Gamma_1' \to 1$, $z + z_0 \to z - z_0$, we arrive at the following expression for the function G^0 characterizing the primary field:

$$G^0(x,z;t) = \left\{0(0 < t < t_0); 1/\sqrt{t^2 - t_0^2}(t_0 < t)\right\} \quad (35)$$

where $t_0 = n_1 R_-$ is the time of arrival of the primary wave at the observation point in the first medium.

V. Field in the Second Medium

Denote the roots of the (25a) and (25b) by η_2^- and η_2^+, respectively. Then the integral in (23) takes the form

$$\begin{aligned} &G^2(x,z;t) \\ &= \frac{i}{2}\lim_{\delta\to 0}\left\{\operatorname*{Res}_{\eta=\eta_2^-}\frac{\Gamma_2''(\eta)}{x\eta + z_0\sqrt{n_1^2-\eta^2} - z\sqrt{n_2^2-\eta^2} - t_-}\right. \\ &\qquad\left. - \operatorname*{Res}_{\eta=\eta_2^+}\frac{\Gamma_2''(\eta)}{-x\eta + z_0\sqrt{n_1^2-\eta^2} - z\sqrt{n_2^2-\eta^2} + t_+}\right\} \\ &= \frac{i}{2}\lim_{\delta\to 0}\left\{\frac{\Gamma_2''(\eta_2^-)}{x - Z(\eta_2^-)} + \frac{\Gamma_2''(\eta_2^+)}{x + Z(\eta_2^+)}\right\} \end{aligned} \quad (36)$$

where

$$Z(\eta) = \left(\frac{z_0}{\sqrt{n_1^2 - (\eta)^2}} - \frac{z}{\sqrt{n_2^2 - (\eta)^2}}\right)\eta \quad (37)$$

$$\Gamma_2''(\eta) = \Gamma_2'(\eta)/\sqrt{n_1^2 - \eta^2}. \quad (38)$$

The expressions for the roots $\eta_2^\pm$ can be written explicitly as the solutions of the associated algebraic quartic equations. However, they are too cumbersome because of six parameters entering (25) and are not used in the present paper. In view of the causality principle, $G^2(x,z;t) \equiv 0$ for $t < t_{tr}$, where t_{tr} is the time of arrival of the transmitted wave at the observation point in the second medium. For $t > t_{tr}$, the roots $\eta_2^\pm$ are complex and, as evident from (25), in terms of (30), we have $\eta_2^- = (\eta_2^+)^*$. Therefore, having regard to (31), we obtain for $t > t_{tr}$

$$\begin{aligned} G^2(x,z;t) &= \frac{i}{2}\lim_{\delta\to 0}\left\{\frac{\Gamma_2''(\eta_2^+)}{x + Z(\eta_2^+)} - \frac{\Gamma_2''(\eta_2^{+*})}{x - Z(\eta_2^{+*})}\right\} \\ &= \frac{i}{2}\lim_{\delta\to 0}\left\{\left[\frac{\Gamma_2''(\eta_2^+)}{x + Z(\eta_2^+)}\right] - \left[\frac{\Gamma_2''(\eta_2^+)}{x + Z(\eta_2^+)}\right]^*\right\} \\ &= -\lim_{\delta\to 0}\mathrm{Im}\frac{\Gamma_2''(\eta_2^+)}{x + Z(\eta_2^+)} \\ &= -\mathrm{Im}\frac{\Gamma_2''(\eta_2^>)}{x + Z(\eta_2^>)} \end{aligned} \quad (39)$$

where $\eta_2^> = \eta_2^+|_{\delta=0}$.

VI. Discussion and Conclusion

Formulas (13) and (35) for the primary field, formulas (32) and (33) for the secondary field in the first medium, as well as formula (39) for the secondary field in the second medium coincide with the relevant expressions derived in [9] by CHM.

The principal result of the work is a new representation for the field generated by a pulsed line current in a two-media configuration in the form of the integrals over a finite contour (20), (21). This method, like the CHM, is applicable to the problems of pulsed electromagnetic radiation from linear sources in media formed by an arbitrary finite number N of homogeneous parallel layers with permittivity ε_j and permeability $\mu_j(j =$

$1, 2, \ldots, N$). In this case, for the field in the layers, the integrals over the contour enveloping the interval $(-n_{\max}, n_{\max})$, where $n_{\max} = \max\{n_1, n_2, \ldots, n_N\}$, are similar to representations (20), (21). Two methods for calculating these integrals are possible.

The first way is to reduce them by the Cauchy theorem to a sum of residues at the poles of the integrand. These poles are determined by the roots of algebraic equations that coincide with the equations for the modified Cagniard contours [9]. Therefore, this technique being alternative to the CHM in an analytical sense is equivalent to it in a calculating sense.

Another way is to estimate numerically the integrals in (20), (21). It is easy to show that they can be reduced to the integrals over the interval $(0, n_{\max})$. For example, the field in the first medium (20) can be represented for $n_2 > n_1, t > t_{ref}$ in the following form:

$$G^1(x,z;t) = -\frac{2}{\pi}t\left\{\int_0^{n_1} f(\eta)\Gamma_1''(\eta)d\eta + \frac{2}{n_2^2 - n_1^2}\int_{n_1}^{n_2} f(\eta)\sqrt{n_2^2-\eta^2}d\eta\right\}$$

where

$$f(\eta) = \frac{x^2\eta^2 + \left(n_1^2-\eta^2\right)(z+z_0)^2 - t^2}{\left(x^2\eta^2 - \left(n_1^2-\eta^2\right)(z+z_0)^2 + t^2\right)^2 - 4x^2t^2\eta^2}$$

$$\Gamma_1''(\eta) = \Gamma_1'(\eta)/\sqrt{n_1^2-\eta^2}.$$

We can use a standard integrating procedure of any mathematical package to calculate G^1 by this formula. Comparison of the data obtained by this way with the explicit expression given by (33) has demonstrated high efficiency and accuracy of this approach.

The key point of the CHM is the solution of the algebraic equation determining the modified Cagniard contour. To do this, iterative numerical methods are used. The greatest difficulty inherent in these methods is to choose the starting value that is close enough to the required zero of the equation [18]. In the paper [9], such an initial approximation has been proposed for the medium consisting of N isotropic layers. For $N = 2$, the efficiency of the iterative method has been shown. For more complex structures containing anisotropic layers, the initial approximation of this kind is unknown. (The CHM allows us to study as yet the simplest situation where the source and the observation point are located on the boundary of an anisotropic medium [19].)

The method proposed in the paper being free from the complications of this kind reduces the calculation of the field generated by a line dipole in a multilayered medium to a standard procedure of numerical integration over a finite interval.

Appendix

Consider a function $\kappa(\omega,\xi) = \sqrt{\omega^2 n^2 - \xi^2}$ in $\mathbf{R^3}$ assuming that the refractive index $n = n' + in''(n', n'' > 0)$ is complex-valued.

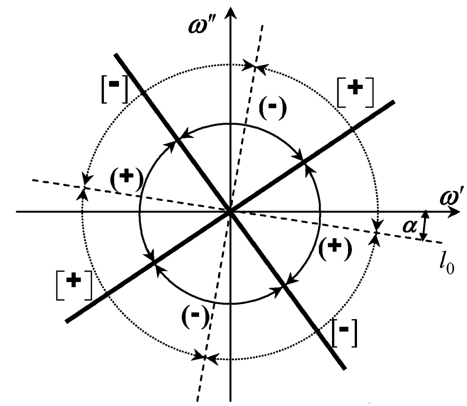

Fig. 3. Sign distribution for $\mathrm{Re}\kappa^2$ and $\mathrm{Im}\kappa^2$ in the plane $\xi' = 0$. Straight lines indicate the lines of intersection with the plane $\xi' = 0$: *a)* bold line—for the cone $\mathrm{Re}\kappa^2 = 0$; *b)* dash line—for the planes $\mathrm{Im}\kappa^2 = 0$. Symbols $(\pm)$ specify a sign of $\mathrm{Re}\kappa^2$, while $[\pm]$ specify a sign of $\mathrm{Im}\kappa^2$; $\sin\alpha = -n''/|n|$, l_0 is the cone axis (A1).

1. A surface

$$\mathrm{Re}\kappa^2 = (n'^2 - n''^2)\omega'^2 - (n'^2 - n''^2)\omega''^2 - 4n'n''\omega'\omega'' - \xi'^2 = 0 \quad \text{(A1)}$$

has the following invariants [16]: $I = -1$, $J = -|n|^4$, $D = -J$, $A = 0$, $A' = D$. Therefore, it represents a two-pole elliptic cone symmetrical with respect to the plane $\xi' = 0$ with its vertex at the origin of coordinates. Let us locate the axis of the cone. The lines of intersection of the cone with the symmetry plane $\xi' = 0$ are two mutually orthogonal straight lines $(n' \mp n'')\omega'' \pm (n' \pm n'')\omega' = 0$ with the bisecting lines $n'\omega'' + n''\omega' = 0$ and $n'\omega' - n''\omega'' = 0$. Consequently, the cone axis is determined by the equations $n'\omega'' + n''\omega' = 0$ and $\xi' = 0$.

2. A surface

$$\mathrm{Im}\kappa^2 = n'n''(\omega'^2 - \omega''^2) + \omega'\omega''(n'^2 - n''^2) = 0 \quad \text{(A2)}$$

has the following invariants: $I = 0$, $D = -|n|^4/4$, $A = 0$. Therefore, it represents two mutually orthogonal planes intersecting along the ξ'-axis and determined by the equations: $n'\omega'' + n''\omega' = 0$ and $n''\omega'' - n'\omega' = 0$. The first plane contains the axis of the cone (A1) being its another symmetry plane. From (A1) and (A2), we derive the following equations for the branch lines of $\kappa(\omega,\xi)$:

$$n'\omega' - n''\omega'' \pm \xi' = 0, \quad n'\omega'' + n''\omega' = 0. \quad \text{(A3)}$$

In Fig. 3, the distribution of signs for $\mathrm{Re}\kappa^2$ and $\mathrm{Im}\kappa^2$ in $\mathbf{R^3}$ is shown.

In (14)–(16), a single-valued branch of the function $\kappa(\omega,\xi)$, for which $\mathrm{Im}\kappa(\omega,\xi) \geq 0$, has been determined on the real plane $P = \{\xi', \omega'\}$. The above mentioned inequality is hold everywhere in $\mathbf{R^3}$ if the following condition is satisfied: $0 \leq \arg\kappa^2 < 2\pi$. In other words, the cut S in $\mathbf{R^3}$ that separates this branch should be determined by the conditions $\mathrm{Re}\kappa^2 \geq 0$, $\mathrm{Im}\kappa^2 = 0$. As is seen from Fig. 3, this takes place for a double sector formed by the intersection of the inner part of the cone (A1) with its symmetry plane $n'\omega'' + n''\omega' = 0$. In $\mathbf{R^3}$ with the cut of this kind (Fig. 4) we have $\mathrm{Im}\kappa(\omega,\xi) \geq 0$.

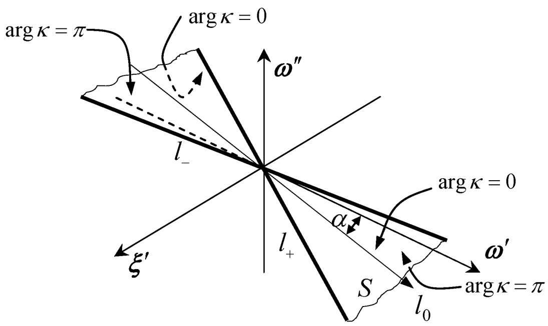

Fig. 4. Location of the branch lines $l_\pm$ and the cut surface S ensuring a choice of the branch for which $\mathrm{Im}\kappa(\omega, \xi) \geq 0$ in $\mathbf{R^3}$-space; l_0 is the cone axis (A1).

A similar approach to choosing a branch of the square root is given in [17] for the case of a single variable. When passing to a lossless medium ($\alpha = 0$), the cut surface S is shifted into the plane $\omega'' = 0$ representing a double sector which contains the ω'-axis and is bounded by straight branch lines $n'\omega' \pm \xi' = 0$.

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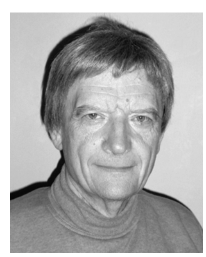

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