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SYNTHESIS OF MODE CONVERTERS IN WAVEGUIDES AND GRATINGS BASED ON SPECTRAL THEORY

Y. K. Sirenko and N. P. Yashina

Institute of Radiophysics and Electronics
National Academy of Sciences of Ukraine
Kharkov, Ukraine

K. F. Schüenemann

Technical University Hamburg-Harburg
Hamburg, Germany

Abstract—The paper demonstrates, relying on spectral theory, how the problem of converter synthesis may be reduced to the search of real eigen-frequencies located in the higher sheets of Riemann's surface for the corresponding eigen boundary value problems (spectral problems). Numerically this problem is solved efficiently by incorporating algorithms, which have been developed for the solution of spectral problems, into the synthesis problem.

1. INTRODUCTION

The efficient synthesis of mode converters, constructed with waveguides and gratings, is on demand in different fields of millimeter wave applications. The major goal of the synthesis problem is to obtain the required results by most efficiently fitting the specifications. The adequacy of the mathematical model of the simulated electromagnetic process is certainly of special importance. Often, for increased efficiency, several parameters and physical features of the process may escape attention. Such a situation appears in simulations of resonant scattering. As an example of the beneficial results that can be obtained from extending the mathematical description of models of periodic and waveguide resonators, the analytic continuation of the diffraction problem into the non-physical domain of complex frequencies can be considered. It is noteworthy that the physical meaning of the mathematical continuation process is not always clear [1–3]. The accurate study of singularities of operators and functions describing scattering processes as functions of complex variables provides the investigators with important information. As is known, the local behavior of a function of complex-valued parameters is determined by its singularities. The implementation of the idea of analytic continuation of a function into the domain of complex parameters is complicated by the necessity to define the continuation of the original diffraction problem into the complex domain of parameters in such a way that it is physically meaningful. Otherwise such non-physical objects as continuous spectra do appear.

This requirement results in a consideration of the 2D problems in such domains of complex variables as multi-sheet Riemann surfaces with logarithm-type branch points in the case of compact scatterers or as algebraic (of the second order) in the case of gratings and waveguides.

The complex amplitudes of the fields in this domain of parameters are meromorphic functions of the complex-valued frequency. This property of the fields and amplitudes opens new perspectives for their qualitative investigation. The spectral theory of open compact, waveguide type , and periodic resonators, developed by the efforts of a considerable number of scientists [1–3], has created a reliable common methodological background for the solution of numerous important mathematical, electromagnetic and applied problems of resonant wave scattering.

The crucial influence of the spectrum elements (that are the singularities of the analytic continuation of the solutions of diffraction problem into the Riemann surface) on the formation of resonance responses of the structure to any (stationary or non-stationary) excitation has in particular been demonstrated. It has resulted in new insights and new ways of treating such electromagnetic features as total reflection and transmission performed by semi-transparent structures, high-Q resonances, complete mode conversion, and others [1, 4]. The bounds of validity of the classical radiation principle, of the limit absorption principle, and of the principle of limit amplitude used for extracting the single physically meaningful solutions of elliptic boundary value problems in non-classic (unbounded) domains are clearly defined [1, 5]. An analytical description of threshold phenomena (Wood's anomalies) in the vicinity of cutoff frequencies of new harmonics in the spatial spectrum of gratings and waveguide discontinuities has been given in [1, 6]. It is noteworthy that exploiting the same scheme of analysis with only a replacement of the complex frequency parameter by a complex distance can provide a mathematically rigorous solution to the problem of determining the field singularities at the edges of a scatterer. These and several other results have been obtained while analyzing the singularities of the analytic continuation of the solution of diffraction problems into the first basic sheet of the Riemann surface.

In this paper, we introduce into consideration the higher sheets. Finding out the real spectral points on these sheets provides a possibility to solve applied problems which are important for resonant quasi-optics, for diffraction electronics, for mm-wave and antenna units, whose operation is based on the conversion of surface waves into spatial waves.

The problem is to determine whether the grating or waveguide with a general cross section is able to perform a complete conversion of homogeneous or non-homogeneous wave packets, with given numbers of components (harmonics or modes), into others that do not coincide with the given packets over all set of numbers of partial components of the spatial spectrum of the structure. If the answer is yes, for what parameter values can this be done.

This problem could hardly be solved by means of traditional methods of diffraction theory. But the solution may be arrived at using the results of a direct investigation of the diffraction characteristics of periodic gratings and discontinuities in multi-mode waveguides, which exhibit the property of complete or close to complete conversion of one propagating wave into another. In the electromagnetic theory of gratings, the complete non-specular wave reflection in the auto-collimating mode or with a large telescope factor [7, 8] may be considered as the most interesting case. In waveguides, the most impressive example may be the phenomenon of strong transformation of the principal mode into a higher one, discovered in simple and complicated waveguide bends [9], coaxial junctions of circular waveguides [10], and others. The search for such regimes has been carried out with efficient algorithms for the solution of diffraction problems, developed on the basis of the semi-inversion method [7, 11], exploiting parametric optimization techniques.

In the present paper, a new approach is suggested for the solution of more complicated problems. It is based on the analysis of the spectrum of the structures under consideration. Complex eigen-frequencies (poles of analytical continuations of the solutions of diffraction problems) of open periodic and waveguide resonators may come to the real axis of either principal or higher sheets of the Riemann surface. In the case of higher sheets, the existence of such real eigen-frequencies determines the possibility of implementing the phenomenon of complete conversion of one wave packet into another [1, 12].

In the first part of the paper, a brief theoretical background of the method is given for a reflecting grating, that is chosen as a model. The principal statements of spectral theory [1] are reformulated in terms of the model considered. The second part demonstrates the efficiency of the suggested method for certain grating and waveguide geometries.

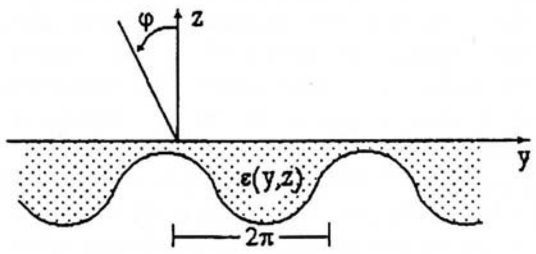

Figure 1. Configuration of the reflective structure.

2. THEORETICAL BACKGROUND OF THE METHOD

Let the grating (see Fig. 1) be excited by a packet of plane E- or H-polarized waves

$$U^i(y,z,\kappa) = \sum_{n\in N^i} a_n e^{i(\Phi_n y - \Gamma_n z)}, \qquad z \geq 0 \tag{1}$$

where N^j is a finite set of integer numbers $\{n\}_{-\infty}^{\infty}$. The determination of the total diffraction field $U^f(g,\kappa)$ can be reduced to the following boundary value problem within the strip (a Floquet channel) $F =$

$\{g: 0 < y < 2\pi\}$. We are seeking the solution $U^f(g,\kappa)$ of the homogeneous Helmholtz equation

$$\left[\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+\kappa^2\varepsilon(y,z)\right]U^f(g,\kappa)=0$$
$$g=\{y,z\}\in Q=F\backslash\text{int}S,\qquad \kappa>0 \tag{2}$$

The solution has to satisfy the radiation condition

$$U^f(g,\kappa)=U^i(g,\kappa)+\infty\sum_{n=-\infty}^{\infty}b_n e^{i(\Phi_n y+\Gamma_n z)},\quad z\geq 0 \tag{3}$$

and the generalized boundary conditions

$$\text{E}_{tg}\big|_{g\in S}=0;\quad \int\limits_{\text{B}}\left(\left|U^f\right|^2+\left|\text{grad}U^f\right|^2\right)dg<\infty \tag{4}$$

$$U^f\left\{\frac{\partial U^f}{\partial y}\right\}(2\pi,z,\kappa)=e^{i2\pi\Phi}\,U^f\left\{\frac{\partial U^f}{\partial y}\right\}(0,z,\kappa) \tag{5}$$

Here $U^{(\ldots)}=E_x$ for the E-polarization $(E_y=E_z=H_x=0)$ and $U^{(\ldots)}=H_x$ for the H-polarization $(\text{E}_y=\text{E}_z=\text{H}_x=0)$; $E_{(\ldots)}$ and $H_{(\ldots)}$ are the components of the vectors of the electric and magnetic field, respectively; $\varepsilon(g)$ is a complex-valued piecewise smooth function (a piecewise constant function for the H-case) characterizing the relative dielectric permittivity of the grating material; S is the piecewise-smooth boundary of the metal components of the grating; int$S$ is the cross section of these components; B is an arbitrary compact in Q, Φ is the real parameter of the Floquet channel F; $\Phi_n=n+\Phi$; κ is the dimensionless frequency parameter characterizing the relation between the grating period and the excitation wavelength. $\Gamma_n=\left(\kappa^2-\Phi_n^2\right)^{1/2}$, $\text{Im}\Gamma_n\geq 0$, $\text{Re}\Gamma_n\geq 0$. The smoothness of the function $U^f(g,\kappa)$ is guaranteed by the required continuity, all over Q, of the tangential components of the field vectors. The time dependence t is assumed to be $\exp(-i\kappa t)$. The radiation condition (3) satisfies the requirement of the absence of the partial components (plane homogeneous and non-homogeneous waves) arriving from $z=\infty$ (the propagation direction is uniquely defined by choosing the root branch and the time dependence of the process), in the scattered field

$$U^s(g,\kappa)=\sum_{n=-\infty}^{\infty}b_n\,e^{i(\Phi_n y+\Gamma_n z)},\quad z\geq 0 \tag{6}$$

The second part of condition (4) limits the field energy in any bounded area and completes the first one near the singular points of the structure geometry.

Let us consider equation (3) in detail, representing the total complete diffraction field in the reflection zone of the structure. The first term is a packet of waves exciting the grating (see (1)). An infinite series determines the scattered (secondary) field (6). The terms of this series are called partial components of the spatial spectrum or diffraction harmonics. Every term of this expansion, when $\mathrm{Im}\Gamma_n = 0$ and $\mathrm{Im}\Gamma_n > 0$, is a homogeneous plane wave outgoing from the grating at an angle

$$\varphi_n^s = -\arcsin\left[\left(n+\Phi\right)/\kappa\right] \tag{7}$$

The plane homogeneous wave packet $U^i\left(g,\kappa\right)$ arrives at the angles of $\varphi_n^i = -\varphi_n^s$ (all the angles are counted in the plane yOz from the axis z anti-clockwise, see Fig. 1). If we represent the amplitudes $b = \left\{b_n\right\}_n$ in terms of a generalized scattering matrix of the structure $R = \left\{R_{np}\right\}_{n,p}$, ($b_n = \sum\limits_{p\in N^i} R_{np}a_p$ or $b = Ra$, $a = \left\{a_n\right\}_{n\in N^i}$) the balance of the scattering harmonic energies (that is analogous to the energy conservation law in the case considered) can be easily derived from the relation [1, 8]

$$\sum_{n=-\infty}^{\infty} \left|R_{np}\right|^2 \mathrm{Re}\Gamma_n = \mathrm{Re}\Gamma_p + 2\mathrm{Im}R_{pp}\mathrm{Im}\Gamma_p - W \tag{8}$$

where the value $W \geq 0$ is determined by the losses in a non-ideal dielectric ($W = 0$ by $\mathrm{Im}\varepsilon\left(g\right) \equiv 0$). In the case of a lossless dielectric, if the grating is excited by a single homogeneous plane wave with an amplitude a_p, the relative energy fraction converted to each scattered field harmonic $U^s\left(g,\,\kappa\right)$ is determined from the value

$$W_{n\rho} = \left|b_n\right|^2 \mathrm{Re}\Gamma_n \Big/ \left|a_\rho\right|^2 \mathrm{Re}\Gamma_\rho$$

Thus we have complete conversion of waves one into another provided that $W_{n\rho} = 1$. Similar conditions of complete conversion can be obtained from equation (8) for any other packet of "incoming" and "outgoing" waves.

The problem described by (2)–(5) can be uniquely solved for each $\kappa > 0$ excluding (at $\mathrm{Im}\varepsilon\left(g\right) = 0$) only a countable set without finite accumulating points [1]. The periodic and waveguide open resonators substantially differ from the compact ones by the presence of such a special point in the physical domain of the frequency parameter variation, which predetermines some uncommon scattering regimes of sinusoidal and non-sinusoidal waves [4].

These special points $\{\bar{\kappa}\}$ are a group of elements of the spectral set Ω (eigen-frequency spectrum), whose complete compound is defined by the characteristic features of the analytic continuation of the solution of the problem described by (2)–(5) from the real axis κ into the domain of the complex values of κ. If continued within the natural boundaries, the analysis domain is extended to the infinite-sheet Riemann surface K consisting of the planes (sheets) κ cut along the lines [1]

$$(\mathrm{Re}\kappa)^2 - (\mathrm{Im}\kappa)^2 - \Phi_n^2 = 0, \quad n = 0, \pm 1, ..., \quad \mathrm{Im}\kappa \leq 0 \tag{9}$$

The branch points κ_m are of second order and are defined by the condition $\Gamma_m(\kappa_m) = 0$, $m = 0, \pm 1, ...$ The first sheet of the surface K ($\Gamma_n(\kappa)$ value distribution), whose real axis is the field of analysis of the diffraction problems (2)–(5), is entirely defined by the conditions

$$\mathrm{Im}\Gamma_n \geq 0, \ \mathrm{Re}\Gamma_n \cdot \kappa \geq 0; \quad n = 0, \pm 1, ..., \quad \mathrm{Im}\kappa = 0$$

which follow from the radiation condition (3) and the cuts (9). The higher-order sheets differ from the first one in as much as signs of $\Gamma_n(\kappa)$ are reversed for a finite number of the n-subscript values.

The function $U^f(g, \kappa)$, is meromorphic in K (see [1]) as a function of the complex variable κ. Its poles $\{\bar{\kappa}_m\}$ form a set, which is discrete in any finite part of K. In the points $\bar{\kappa}_m \in \Omega$, the homogeneous problem

$$\left[\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \kappa^2 \varepsilon(g)\right] U(g, \kappa) = 0, \quad g \in Q, \quad \kappa \in K \tag{10}$$

$$U(g, \kappa) = \sum_{n=-\infty}^{\infty} c_n e^{i(\Phi_n y + \Gamma_n z)}, \quad z \geq 0, \quad \kappa \in K \tag{11}$$

$$\mathrm{E}_{tg}\big|_{g \in S} = 0 \tag{12}$$

$$U\left\{\frac{\partial U}{\partial y}\right\}(2\pi, z, \kappa) = e^{i2\pi\Phi}\, U\left\{\frac{\partial U}{\partial y}\right\}(0, z, \kappa), \quad \kappa \in K \tag{13}$$

(see also (2)–(5) can be solved in a non-trivial way. Its solutions $U(g, \bar{\kappa}_m)$, $\bar{\kappa}_m \in \Omega$ are called the natural field oscillations in the open periodical resonator (in the grating), and the corresponding complex numbers $\bar{\kappa}_m$ are the eigen-frequencies.

Assume here that, at certain values of the independent variables, one of the complex eigen-frequencies $\bar{\kappa}$ crosses the real axis within the strip $\mathrm{Re}\kappa > 0$ of one of the higher (non-physical) sheets of the surface K. Let us analyze the partial components of the natural oscillation field (11) in the reflection zone of the structure. A part of them have numbers n such that $\bar{\kappa} < |\Phi_n|$ are non-homogeneous plane waves which decrease (if $\mathrm{Im}\Gamma_n(\bar{\kappa}) > 0$) or increase exponentially (if $\mathrm{Im}\Gamma_n(\bar{\kappa}) < 0$) with increasing z. The set of partial field components (11) in addition to the first set having numbers n such that $\bar{\kappa} > |\Phi_n|$, merges with the homogeneous plane waves arriving at the grating (in the case of $\mathrm{Re}\Gamma_n(\bar{\kappa}) < 0$) or going out of it (in the case of $\mathrm{Re}\Gamma_n(\bar{\kappa}) > 0$).

Then we form two packets, $U_1(g, \bar{\kappa})$ and $U_2(g, \bar{\kappa})$, from all the partial components of the natural oscillation field with eigen-frequency $\bar{\kappa}$ and $\mathrm{Im}\bar{\kappa} = 0$ which do not intersect on the set $\{n\}_{-\infty}^{\infty}$:

$$U_j(g, \bar{\kappa}) = \sum_{n \in N_j} c_n e^{i(\Phi_n y + \Gamma_n z)} \tag{14}$$

Here, $N_1 + N_2 = \{n\}_{-\infty}^{\infty}$, i.e.

$$U_1(g, \bar{\kappa}) + U_2(g, \bar{\kappa}) = U(g, \bar{\kappa})$$

$$N_1 = \{n : \mathrm{Im}\Gamma_n(\bar{\kappa}) < 0 \quad \text{or} \quad \mathrm{Re}\Gamma_n(\bar{\kappa}) < 0\ \}$$
$$N_2 = \{n : \mathrm{Im}\Gamma_n(\bar{\kappa}) > 0 \quad \text{or} \quad \mathrm{Re}\Gamma_n(\bar{\kappa}) > 0\ \}$$

We can represent the fact of the occurrence of the natural field oscillations in the grating at a real eigen-frequency $\bar{\kappa}$ of one of the higher sheets of the surface K in terms of the general diffraction problem (2)–(5). As follows from the representation (14) for $U_j(g, \bar{\kappa})$, while exciting the grating by a plane wave packet $U_1(g, \kappa)$ at frequency κ, which coincides with the projection of the point $\bar{\kappa}$ onto the first sheet of the surface K, the secondary field is only determined by the plane wave packet $U_2(g, \kappa)$, and the natural oscillation field $U(g, \bar{\kappa})$ overlaps with the full diffraction field

$$U^f(g, \kappa) = U(g, \kappa) = U_1(g, \kappa) + U_2(g, \kappa)$$

Thus the eigen-frequency $\bar{\kappa}$ determines the excitation frequency κ where one plane wave packet is completely converted into another one by the grating; the packets overlap in none of their components (i.e. they contain different harmonics of their spatial spectrum). This means that the solution of the problem of complete conversion can be reduced to searching for the real eigen-frequencies $\bar{\kappa}$ lying on the higher sheets of the Riemann surface K. The latter problem is solved by employing numerical algorithms. Examples of several of the algorithms are given in papers [1, 2] and [13]. The characteristics of the packets U_j entirely determine the Riemann surface sheet and a part of the real axis between two nearest branching points which is the domain of the search. The sheet is defined by the signs of the real and imaginary parts of the propagation constants $\Gamma_n(\kappa)$ in the algorithm of solving the spectral problem (10)–(13), and the part of the real axis is determined by the number k such that $\min\{/\text{Re}\Gamma_n(\kappa)/\}$ of the components of both packets U_j is reached at $n = k\,(\kappa > |\Phi_k|)$. Remember one restriction which is caused by the specificity of the spectral problems (see [1]): the number of waves forming the set U_1 must be finite.

According to theorem 1.10 from [1], there is one more sheet of the surface K on which the projection of the point $\bar{\kappa}$ from the sheet under consideration is a spectral point as well (For convenience, the theorems and lemma mentioned in the text are given in an Appendix). Here, the real parts of the propagation constants Γ_n of the homogeneous plane waves, which are the partial components of the natural oscillation field (11), reverse their signs. This means that the point κ of the frequency band, which is a projection of the eigen-frequency $\bar{\kappa}$ onto the first sheet of K, is the point at which there is a complete conversion of the wave packet $V_1(g,\kappa)$ into the wave packet $V_2(g,\kappa)$, which contains all the homogeneous waves of the packet $V_1(g,\kappa)$ $U_2(g,\kappa)$ (only the harmonic numbers, not the amplitudes) and all the non-homogeneous waves from $U_1(g,\kappa)$, and $V_2(g,\kappa)$ contains all the homogeneous waves from $U_1(g,\kappa)$ and the non-homogeneous ones from $U_2(g,\kappa)$. Lemma 1.5 from [1] (see Appendix) and the reciprocity relation [7] enable us to correlate the above case with some other situations by substituting the constant Φ with $(-\Phi)$ under the quasi-periodicity conditions of the excitation problem (the change to the structure "rotated" around the z-axis).

The problems of complete conversion of the wave packets containing several propagating harmonics can hardly be efficiently solved by the diffraction theory methods (i.e. as direct problems). In view of this, the advantages of the spectral approach are irrefutable. This approach is applicable under any conditions and provides complete information on the scattering process: on the total diffraction field (natural oscillation field); on the packet component amplitude $U_j\,(g,\kappa)$ (the amplitudes of the partial components of the natural oscillation field in the radiation zones); on the operation frequency (the eigen-frequency $\bar{\kappa}$), and on the structural parameters of the scattering regime.

3. NUMERICAL RESULTS

The first example is the problem of natural oscillations of an E-polarized field in a reflection grating with rectangular lamellas and dielectric slot filling (see Fig. 2b: $\theta = 0.8$, $\varepsilon = 3.89$, $\Phi = 0.1$; $2\pi\delta$ is the profiling depth of the structure, $2\pi\theta$-width of slots). Fig. 2a

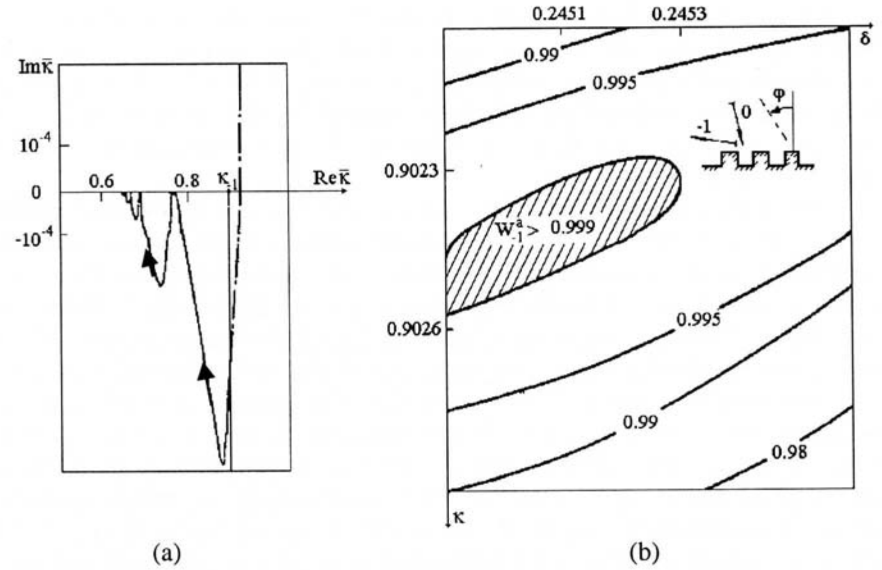

Figure 2. Results of applying the spectral (a) and the classical (b) approach to searching for the regime of complete wave conversion (E-polarization, $\theta = 0.8$, $\varepsilon = 3.89$, $\Phi = 0.1$) for the diffraction problems (2)–(5).

represents one of the spectral curves $\bar{\kappa}(\delta)$, the arrows on curve $\bar{\kappa}(\delta)$ show the direction of δ increasing. A part of the curve $\bar{\kappa}(\delta)$, indicated by the solid line, shows the variation of the eigen-frequency value in the first, physical, sheet of the surface K. The most important points here are those of $\bar{\kappa}(\delta)$ that are lying on the real axis; the Q-factor of the natural oscillation increases without any limit (this is the H_{021}-type oscillation: the resonance at the H_{02}-wave in the slot with one complete field variation along the structure depth). The branch cut starting in the threshold point of the first harmonic ($\kappa = \kappa_{-1}$) separates the physical and the non-physical sheets of the surface K. The cut is shown in Fig. 2a by a wavy line.

With δ decreasing, the element κ of the spectral set Ω moves to the second sheet (dashed continuation of the curve) and crosses its real axis in the point $\kappa = 0.9024$ with $\delta = 0.245$. Over this part of the real axis, $\mathrm{Re}\Gamma_{\circ} > 0$, $\mathrm{Re}\Gamma_1 < 0$, $\mathrm{Im}\Gamma_{\circ} = \mathrm{Im}\Gamma_{-1} = 0$, and for the rest $n \neq 0, -1$ holds $\mathrm{Re}\Gamma_n = 0$ and$\mathrm{Im}\Gamma_n > 0$. According to the conclusions made in part II, the grating with parameters $\delta = 0.245$, $\theta = 0.8$, $\varepsilon = 3.89$, the Floquet channel parameter $\Phi = 0.1$, and the frequency parameter $\kappa = 0.9024$ must completely convert the first incident harmonic into

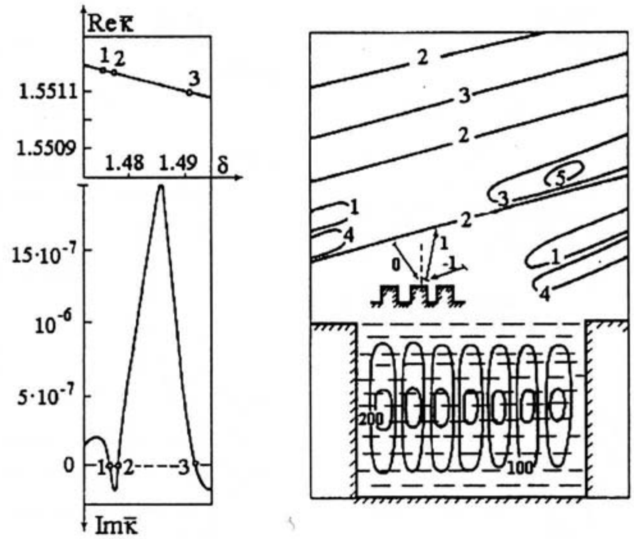

Figure 3. Spectral analysis of the effects of the complete wave packet conversion (E-polarization, a grating with dielectric-filled slots, $\theta = 0.8,\ \varepsilon = 8.0,\ \Phi = 0.33$).

a zero outgoing one and vice versa. In this case, $\varphi^{i}_{-1} = -85.8°$ and $\varphi^{s}_{\circ} = -6.4°$ or $\varphi^{i}_{\circ} = -6.4°$ and $\varphi^{s}_{-1} = 85.8°$.

Fig. 2b is, in fact, a confirmation of the predicted effect. The figure shows, in the coordinates κ, δ, the lines of equal level of the relative part of the energy $W_{-1.0}\ (\kappa, \delta) = \text{const}$ converted to the first propagating harmonic of the field $U^{s}\,(g, \kappa)$, if the structure considered is excited by the principal (zero) wave of the spatial spectrum. As expected, the area of the almost complete conversion with $W_{-1,0} > 0.999$ covers the point $\{\kappa = 0.9024,\ \delta = 0.245\}$, which has been found by the spectral method.

We have considered above the result of using the spectral method for analyzing the conversion properties of gratings in one of the simplest situations when each of the packets $U_j\ (g, \kappa)$ contains only one homogeneous plane wave. In the cases listed in Figs. 3 and 4 and in the table, the packet U_1 contains only homogeneous waves (one to three), and the packet U_2 contains an infinite spectrum of non-homogeneous waves and one to three homogeneous waves. The line diagrams demonstrate the spectral lines $\mathrm{Re}\bar{\kappa}\,(\delta)$ and $\mathrm{Im}\bar{\kappa}\,(\delta)$ lying on the respective sheets of the Riemann surface K. The sheet is defined by the sign distribution at $\mathrm{Re}\bar{\kappa}\,(\delta)$ in the situation described as "direct" in the table. An inverse situation is associated with the

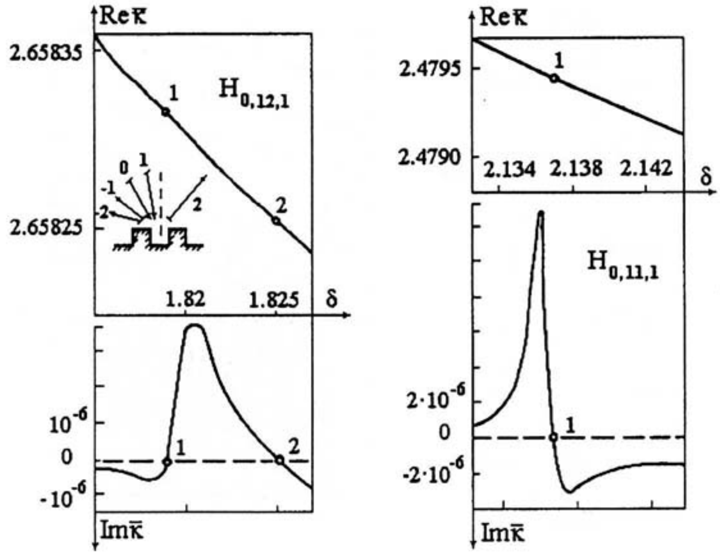

Figure 4. The same as in Fig. 3, but for different wave packets U_1 and U_2.

conversion of the homogeneous waves from one packet into another. The points where $\text{Im}\bar{\kappa}(\delta) = 0$ are marked with appropriate numbers. Fig. 3 demonstrates a fragment of the distribution of the eigen field $|U(g, \bar{\kappa})| = \text{const}$ (or, which is the same, but expressed in terms of the general diffraction problem (2)–(5), the lines of the equal level of the total diffraction field $|U^f(g, \kappa) = \text{E}_\kappa^f(g, \kappa)|$) on a single period of the grating taken in point 3 of the spectral curve. Complete information on the regime of the wave packet conversion in the point where $\text{Im}\bar{\kappa}(\delta)$ vanishes is given in the Table. Here the values κ, δ are presented with their first three decimals only.

The spectral method allows us to synthesize the gratings supporting additional interesting scattering regimes. One of them is described below.

The packet $U_1(g, \kappa) = U^j(g, \kappa)$ consisting of non-homogeneous waves is converted at frequency κ into the packet $U_2(g, \kappa) = U^s(g, \kappa)$ consisting of non-homogeneous waves, too, provided there is an eigen-frequency $\bar{\kappa}$ (the projection of $\bar{\kappa}$ to the first sheet coincides with κ) lying on the Riemann surface sheet which is determined by reversing the signs of Γ_n, $n \in N_1$ of the first sheet. It should be noted that if free space radiation of the propagating harmonics is possible at this κ, then, according to theorem 1.3 from [1] (see Appendix A)

Table

Figure	Oscillation mode	Point where $\mathrm{Im}\overline{\kappa}(\delta)=0$	Situation	Amplitudes of the incident homogeneous waves	Amplitudes of the outgoing homogeneous waves	Parameters of the complete conversion regimes κ	δ
3	$H_{0,7,1}$	1	Direct	$a_1 = 1,5 + i2,8$	$b_0 = 2$ $b_{-1} = -0,51 - i1,1$	$1,551$	$1,476$
			Inverse	$a_0 = 2$ $a_{-1} = 0,88 + i0,82$	$b_1 = 3,1 + i0,61$		
		2	Direct	$a_1 = 3,08 + i0,56$	$b_0 = 2$ $b_{-1} = 1,1 - i0,2$	$1,551$	$1,477$
			Inverse	$a_0 = 2$ $a_{-1} = 0,53 - i0,99$	$b_1 = 1,48 + i2,75$		
		3	Direct	$a_1 = -2,81 + i3,6$	$b_0 = 2$ $b_{-1} = 0,1 - i2,74$	$1,551$	$1,491$
			Inverse	$a_0 = 2$ $a_{-1} = 2,64 + i0,75$	$b_1 = 2,6 - i3,8$		
4,a	$H_{0,12,1}$	1	Direct	$a_0 = 2$ $a_1 = 2,9 + i1,7$	$b_{-1} = 1,4 + i0,35$ $b_{-2} = 0,77 + i3,3$ $b_2 = -1,7 - i1,7$	$2,658$	$1,819$
				$a_{-1} = 0,09 - i1,4$ $a_{-2} = -2,6 + i2,2$ $a_2 = 0,33 - i2,4$	$b_0 = 2$ $b_1 = 2,5 + i2,2$		
		2	Direct	$a_0 = 2$ $a_1 = -0,24 + i2,1$	$b_{-1} = 2,3 + i0,58$ $b_{-2} = -0,56 - i1,3$ $b_2 = 0,79 - i0,73$	$2,658$	$1,825$
			Inverse	$a_{-1} = 0,16 - i2,4$ $a_{-2} = 1,2 - i0,74$ $a_2 = 1,1 - i0,13$	$b_0 = 2$ $b_1 = 1,9 - i0,86$		
4,b	$H_{0,11,1}$	1	Direct	$a_0 = 2$ $a_1 = -3,9 - i0,86$	$b_{-1} = 1,4 - i3,3$ $b_{-2} = 0,05 + i2,5$ $b_2 = -0,55 + i0,56$	$2,479$	$2,137$
			Inverse	$a_{-1} = 3,6 - i0,33$ $a_{-2} = -1,5 + i1,9$ $a_2 = 0,78 + i0,14$	$b_0 = 2$ $b_1 = -2 - i3,4$		

the amplitudes of these harmonics in the natural oscillation field: $U\ (g, \bar{\kappa}) = U_1\,(g, \bar{\kappa}) + U_2\ \ (g, \bar{\kappa})$, and hence also in the total diffraction field: $U^f\,(g, \kappa) = U_1\,(g, \kappa) + U_2\ \ (g, \kappa)$ must become zero.

The significance of investigating this situation in many applied problems becomes more evident, if one takes into account that the surface eigen waves of the planar dielectric waveguides (the E-polarization) or the eigen field of the density-modulated electron beam (the H-polarization) can be used as components of the packet $U_1\ \ (g, \kappa)$.

4. MAIN RESULTS AND CONCLUSIONS

The spectral theory of open periodical and waveguide resonators studies the characteristic features of the analytic continuation of the resolvents of the stationary boundary value problems of type (2)–(5) into the area of the complex (non-physical) values of one of the parameters (the spectral parameter). The theory suggests a common methodology for solving many actual mathematical, physical, and applied problems of the theory of electromagnetic wave resonance scattering. In this paper, the methods and results of the spectral theory (the methods of solving direct problems) are used by solving rather specific inverse problems of scattering theory. Periodical structures of specified configurations were synthesized, which implement the qualitatively specified scattering regimes. For demonstration reflection gratings were chosen, but there is no principal difference in the approach if one considers semi-transparent, selectively reflecting, and transmitting gratings.

The analysis of waveguide resonators as semi-transparent structures is even more interesting with regard to applications, than an analysis of periodic structures (gratings). A free selection of the quantity, configuration, and the spatial orientation of the channels feeding and carrying out the electromagnetic field energy, offers to the experimenters unlimited possibilities of choosing the regimes, and presents the theoreticians with a lot of non-standard problems including the ones of complete mode packets conversion.

The general statement of the spectral boundary value problems and the main conclusions from the analysis of their solutions on different sheets of the Riemann surface for waveguide open resonators are essentially the same as the above. The specificity (e.g., the alternation of the branching points and their location) is caused by the obvious changes in the radiation conditions of type (11), which are to be formulated for each individual semi-infinite channel open for energy transmission [3]. Solution of the problem for waveguides can be

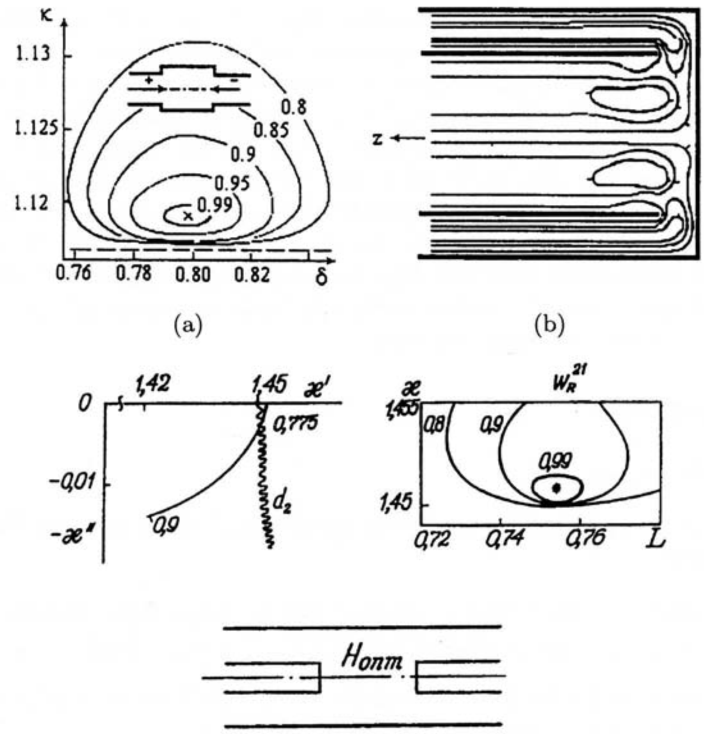

Figure 5. Complete wave conversion in waveguide open resonators.

illustrated by two simple examples of a full conversion of wave modes at the circular waveguide widening (Fig. 5a) and the coaxial-waveguide turn of a circular waveguide, with an infinitely thin inner wall (Fig. 5b; θ is the ratio of the radii of the narrow and the wide waveguide; δ is the resonator length normalized to the wide waveguide radii b; $\kappa = b/\lambda$).

For the widening of a circular waveguide with $\theta = 0.8$, one of the eigen modes with field distribution $\mathrm{E}_y(r, z)$ (r and φ are the polar coordinates in the cross section), which is antisymmetric along the longitudinal axis z, has a real eigen-frequency $\bar{\kappa} = 1.119$ at $\delta = 0.798$. According to the radiation conditions for this eigen mode, the H_{02}-waves of the semi-infinite waveguides are the incoming ones, and the H_{01}-waves are the outgoing ones. Fig. 5a shows the lines of equal value of the power coefficient conversion of the opposite-phase $H_{01}\,(H_{02})$ waves arriving from the two channels of the circular waveguide widening into the reflected $H_{02}\,(H_{01})$ waves: $W_{21}(\kappa, \delta) = W_{12}(\kappa, \delta) = \text{const}$. At the point with coordinates $\kappa = \bar{\kappa}$, $\delta = \bar{\delta}$ (its location is marked with an asterisk), $W_{21} = W_{12} = 1.0$, which is in full agreement with the results of spectral problem analysis.

Fig. 5b demonstrates the distribution of the lines $E_y(r,z) = \text{const}$ of the total diffraction field in a coaxial-waveguide turn whose parameters $\theta = 0.65$, $\delta = 0.297$, $\kappa = 0.938$ provide a complete conversion of the H_{01}-wave of a narrow circular waveguide into the H_{01}-wave of the coaxial waveguide. This set of parameters has been found from the solution of the corresponding spectral problem with radiation conditions that the H_{01}-wave of the coaxial waveguide is the incoming one, and the other waves in both channels of the coaxial-waveguide turn are outgoing ones.

APPENDIX A.

A.1. Theorem 1.3

If $\text{Im}\varepsilon(g) = 0$, then for volumetric gratings ($\delta \neq 0$) we have following statements:

- for $\text{Im}\bar{\kappa} \neq 0$ and $\text{Re}\bar{\kappa} \neq 0$ there are no such eigen modes of field for which the relation $\text{Re}\bar{\kappa}\text{Im}\bar{\kappa}\sum\left(|a_n|^2 + |b_n|^2\right)\text{Re}\Gamma_n > 0$ is valid;
- for $\text{Im}\bar{\kappa} = 0$ and $\text{Re}\bar{\kappa} \neq 0$ there are no such eigen modes of field for which the relation $\sum\left(|a_n|^2 + |b_n|^2\right)\text{Re}\Gamma_n \neq 0$ is valid;
- for $\text{Im}\bar{\kappa} \neq 0$ and $\text{Re}\bar{\kappa} = 0$ there are no such eigen modes of field for which the relation $\text{Im}\Gamma_n > 0$ for all $n = 0, \pm 1, \pm 2, ...$ is valid;
- for $|\text{Im}\bar{\kappa}| \geq |\text{Re}\bar{\kappa}|$ there are no such eigen modes of field for which the relation $\sum\left(|a_n|^2 + |b_n|^2\right)\text{Im}\Gamma_n \geq 0$ is valid.

A.2. Theorem 1.10

Suppose that the point $\bar{\kappa} \in \Omega_\kappa$ with $\text{Im}\bar{\kappa} < 0$ (damped in time eigen mode of the field) does exist. Then the sheet of multi-folded surface K with point $(-\bar{\kappa})$ such that $-\bar{\kappa} \in \Omega_\kappa$ in it can be found (increasing with time eigen field). Ω_κ is a set of eigen values.

A.3. Lemma 1.5

If $\text{Im}\varepsilon(g) = 0$, then $G(g, g_0, \kappa, \Phi) = G(g_0, g, \kappa, -\Phi) = G^*(g, g_0, -\kappa^*, -\Phi)$. Here $G(g, g_0, \kappa, \Phi)$ is the Green's function of the grating.

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Yury K. Sirenko Received Candidate and Doctor of Science degrees in Physics and Mathematics from Kharkov State University, Ukraine, in 1978 and 1988, respectively. Since 1974 he has been the staff of the Institute of Radiophysics and Electronics of National Academy of Sciences of Ukraine as post-graduate student, researcher, senior researcher, and head of the Department of Wave Scattering Theory. Since 1989 he has been a Professor of Mechanics and Mathematics Faculty of Kharkov State University. His current research activities are focused on mathematical simulation in applied electromagnetics.

Nataliya P. Yashina received a Candidate of Science degree in Physics and Mathematics from Kharkov State University, Ukraine, in 1979. In 1973 she joint the Department of Mathematical Theory of Diffraction in the Institute of Radiophysics and Electronics of National Academy of Sciences of Ukraine as graduate student. Now she is a senior researcher of the Institute. In 1996–1998 she was with Gebze Institute of Technology, Turkey as associate professor. Her interest are in analytical and numerical methods of diffraction theory, resonant scattering in waveguides.

Klaus Schünemann was born in Braunschweig, Germany, in 1939. He is Professor of Electrical Engineering and (since 1983) Director of the "Arbeitsbereich Hochfrequenztechnik" at the Technical University of Hamburg-Harburg, Germany. His research interests are transport theory, physical modeling and simulation of submicron devices, analysis and design of electron beam devices, millimeter wave semiconductor power generation and combining, quasi-optical and planar circuit technologies, components for satellite communications, and industrial applications of microwaves.